THE ECONOMICS

OF

STATIONARY STATES

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BY

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MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON
1935

PRINTED IN GREAT BRITAIN
BY R. & R. CLARK, LIMITED, EDINBURGH

PREFACE

In one sense this book is merely an introductory prelude to economics: for it has nothing to say of processes of change or conditions of disequilibrium. But over the field that it covers it aims at being thorough. Starting with very simple conditions and thereafter introducing successive complications, it builds up a picture of the way in which fundamental economic forces work themselves out. These forces are, of course, no less present in a moving than in a stationary world. The subject-matter of the book is thus not an unreal abstraction; it is a limited part or aspect of the real. The text contains no technical apparatus, but some of the issues which are there discussed in ordinary language are worked out more fully with the help of algebraic analysis in Appendices. My best thanks are due to Mr. R. F. Kahn of King's College, Cambridge, for reading the whole of a preliminary draft in manuscript and making a number of valuable criticisms; to Mr. D. Champernowne, also of King's College, for help with parts of Appendix VII; and to Mr. N. J. Leather of the London School of Economics, for revising all the Appendices.

A. C. P.

King's College,
Cambridge, *April* 1935

TABLE OF CONTENTS

CHAPTER XLIII

PAGE

CHAPTER XLIV

CHAPTER XLV

CHAPTER XLVI

CHAPTER XLVII

CHAPTER XLVIII

APPENDICES

CHAPTER I

INTRODUCTORY

§ 1. In the investigations of biologists into non-human life the standpoint taken is almost wholly objective. It is found that the newly hatched bee for the first ten days of its existence devotes itself exclusively to caring for the young inside the hive, for the next ten to building, cleaning and guarding the hive itself, and for the remainder to collecting pollen and nectar from outside.[1] These things happen. The way that they happen is observed and described, and the various responses that are made to visual, odorous and chemical stimuli are set out. But nothing is said about the bee's consciousness. Whether it enjoys collecting honey, whether it experiences a sense of duty well done when it drives away an intruder, we do not know, and, what is more to the present purpose, we need not know. The study of bees is behaviouristic; it is concerned with what they do. It can be carried through successfully without any reference to thought or feeling, and without any need to inquire whether such a thing as thought or feeling exists at all. It is open to us, if we so choose, to study human society on a like objective plan. Certain price and cost situations are found to be associated with certain types of activity and certain distributions of the working population among different avocations. A complete description of everything that happens can be given without mention of the fact that human beings think and feel. We find that, at a given set of prices, they spend more money on one thing than on another, and that, as the price system changes, the relative proportions of money spent on different

[1] Wells and Huxley, *The Science of Life*, p. 1712.

things change in a given way. We need ask nothing about their states of mind. Whether there are such things or not is immaterial; whether, if there are such things, they are causally effective or mere epi-phenomena of chemical or electrical reactions are questions outside the province of objective behaviouristic science.

§ 2. There is no great difficulty in constructing a system of economics on this plan. But there is not, I think, much to be said for doing so. We know, as a matter of fact, that, among human beings, states of mind do exist and that conduct is regulated in accordance with them. It may be, no doubt, that they are not the causes of what we do, but the joint effect, with what we do, of a hidden chemistry. But, however readily we may admit this possibility in our philosophic moments, we inevitably in ordinary life look upon states of mind as genuine causal factors. When we find motor cars being used instead of horses and carts, and are told that people, having learnt how to make motor cars, prefer them to horses and carts, we feel that the situation has been "explained", not indeed completely, but, nevertheless, in a much more satisfactory way than would have been possible if all mention of human consciousness had been held taboo. As Seignobos has well said: "Material objects are never more than the occasion or condition of economic facts; the true economic facts are the ideas of men in relation to these objects".[1] In this book, therefore, the discussion will not be objective in the rigid sense of the preceding section. The economic cosmos will rather be regarded as an outflowing of the human spirit; as something that we build, at least in part, by conscious process, and that is understood only when it has been related to that process.

§ 3. Granted then that states of mind may properly come into our story, with what body should they come? In modern physics it is the rule to extrude from the citadel of science everything that is not in principle capable of measurement by instruments. If this rule is to be followed in economics, desires and aversions as such must not be invoked. As Tertullian said of the demons that possess certain unfortunates

[1] *La méthode historique appliquée aux sciences sociales*, p. 13.

among mankind, "Invisible and intangible, these spirits are not visible in the act; it is in their effects that they are frequently observed". The effects of desires and aversions are schemes of preference and indifference. The quantity of X that a particular man would be just willing, in given circumstances, to surrender in exchange for a given quantity of Y is, in principle, capable of being observed and measured. On the physical analogy it is by these schemes of preference and indifference that, in our studies, states of mind must be represented. We should not attempt to go behind them to the desires and aversions out of which, as introspection tells us, these schemes spring. There is an aesthetic satisfaction in this austerity. Desires and aversions in their own persons cannot be worked into the body of our analysis unless they are treated as quantities. Since they are wholly subjective, it is inconceivable that an instrument can ever be designed that will measure them themselves, as distinct from their effects. Consequently, experimental warrant for treating them as quantities is, in the nature of things, unattainable. Nevertheless, I am loath to let them go. Nor, as I understand the matter, is there any logical compulsion on me to do so. It is true that, if we start from scales of preference, we cannot draw inferences about the *absolute* intensities of desires and aversions; only about the ratio of these intensities at the margin. But, if we start with desire attitudes and aversion attitudes, we can infer, without any ambiguity, associated scales of preference. It is open to us, therefore, if we choose, to start there. In the first part of my analysis, chiefly in Chapters VIII.-X., when the issue arises, I do choose to do this. No difference is made to the substance of the argument by this decision. Anyone who wishes to do so can easily translate what is said into the language of objective preference scales.[1] With this understanding he may not be unwilling to allow me to follow English and French tradition.

§ 4. In the spirit of this tradition Cournot describes the urge behind economic action thus: "We shall invoke but a single axiom, or, if you prefer, make but a single hypothesis,

[1] Cf. Hicks, "The Theory of Value Reconsidered", *Economica*, February 1934.

i.e. that every one seeks to derive the greatest possible value from his goods and labours".[1] Again in like manner Edgeworth writes: "Economics investigates the arrangements between agents each tending to his own maximum utility".[2] It is tempting to translate these descriptions of the fundamental economic urge by saying simply that it consists in an effort on the part of every agent to maximise his satisfaction. This language, however, tacitly assumes that desires and aversions depend upon and are proportionate to the satisfaction or dissatisfaction anticipated from the presence of the desired or repelled object. There is no call here to make that assumption. We may say for brevity that the active principle in building the economic cosmos is an urge on the part of every agent to "maximise his satisfaction"; but we must define maximising his satisfaction to mean so acting that a more keen desire is always followed in preference to a less keen one, and that, when a desire can only be satisfied at the expense of undergoing an aversion, it will be followed or not according as it or the aversion is the keener. The task of positive economics, as distinguished from welfare economics, is to display the structure and working of the economic cosmos as an outgrowth of that principle. This volume is an essay in positive economics.

§ 5. As the title implies, its scope within that field is narrowly limited. The actual economic world, as everybody knows, is always undergoing highly complex forms of movement. It is distinguished by two dominant characteristics. On the one hand, the fundamental governing conditions, the number of the people, their age distribution, their tastes, their stock of equipment, and, last but not least, their endowment of scientific and technical knowledge are constantly changing. On the other hand, the adjustments that are made to these changes accomplish themselves, not by an instantaneous process, but gradually; for knowledge percolates slowly, and, having percolated, only slowly overcomes the inertia of habit. Consequently, the state of things that *tends* to be established

[1] *Theory of Wealth*, Bacon's translation, p. 44.

[2] *Mathematical Psychics*, p. 6. Cf. Moore, "Paradoxes of Competition", *Quarterly Journal of Economics*, 1906, p. 313.

is different at one time from what it is at another, and the state that is actually established at any time diverges in varying degrees from the state which at that time tends to be established. From these complexities there emerge movements of long period trend, movements of cyclical oscillation and movements associated with the procession of the seasons within the year. A full discussion of positive economics would, of course, seek to comprehend the functioning of the maximum principle in this ever-changing concrete actuality. But the problem is too complex for direct frontal attack. "In all negotiations of difficulty", Bacon wrote, "a man may not look to sow and reap at once, but must prepare business and so ripen it by degrees." In this spirit economists have devised and made use of the fiction of a stationary state. Nobody pretends that the study of this artificially simplified model is of value for its own sake. But, as an introductory stage towards understanding real conditions, it has an important part to play. For in our imagined stationary state, no less than in the actual world, the maximum principle is at work; and the very fact that here the environing conditions are moderately simple enables us to grasp its significance with sureness and precision. It is true, as Marshall has pointed out, that "nearly all the distinctive features of a stationary state may be exhibited in a place where population and wealth are both growing, provided they are growing at about the same rate, and there is no scarcity of land: and provided also the methods of production and the conditions of trade change very little; and above all where the character of man himself is a constant quantity".[1] But, since this conception stands but little nearer to reality, while it is less easy to handle, than that of a stationary state proper, the simpler model may reasonably be preferred. This is my excuse, if excuse be needed, for publishing a book of this character.

§ 6. It will have been noticed that in the title stationary states, not a stationary state, are spoken of. This is to indicate that we shall not be concerned merely with the formal conditions that are implicit in the fact of stationariness, but that the differences, which are associated with different sets

[1] *Principles*, p. 368. Cf. *post*, Appendix I.

of governing conditions, are also brought into account. Thus, if we imagine ourselves starting from one stationary state, into which some new element is introduced, a comparison between this stationary state and the new stationary state, that would eventually result if and when full adjustment had been made to the change, falls within our purview. But all processes of transition, everything that is roughly included under the term short-period problems, are excluded. This, as will appear in the sequel, dispenses us from anything beyond a very summary treatment of monetary problems; for, in abstracting from processes of change, we have quitted the terrain on which money offers difficulties to analysis.

§ 7. The scheme of the book is as follows. In Chapters II. to IV. the general character of a stationary state is discussed, and a distinction is drawn between the thorough-going form and modified forms of it. It is decided to concentrate attention upon the thorough-going form. After treating in Chapters V. to VII. of some preliminary matters, I proceed in the next three chapters to study a Robinson Crusoe economy containing one man but many commodities. In the first two of these chapters Robinson is supposed to work without the aid of accumulated capital. In the last of them capital and the rate of interest are brought into account. In Chapters XII. and XIII. transition is made from a Crusoe economy to one that embodies exchange and co-operation in production: and this is followed in Chapter XIV. by a brief account of the part played by money in such an economy. After a chapter on the definition and implications of markets, I begin in Chapter XVI. an analysis of some important problems of exchange in a model made simple by the assumption that exchange is not accompanied by co-operation in production. In Chapters XVI. to XIX. the conditions and implications of competition and various sorts of monopoly are reviewed; and in Chapters XX. and XXI. the conditions of equilibrium in a stationary state and the difference between two stationary states in an exchange economy of the simple kind so far assumed are studied. After this the argument passes to a third model, in which co-operation in production is brought into account; but this

time it is supposed that there is only a single kind of finished commodity. Chapters XXII. to XXXIV. examine this model at length on the assumption that costs of transport between different places are nil, while Chapter XXXV. sets out the consequences of removing that assumption. In my fourth and final model the characteristics of the last two are combined; that is to say, there is both exchange among commodities and services and co-operation in production. In the preceding discussion much of what is relevant here has, of course, already been said. But in the more complex model there are also important features which were not present in either of the two previous ones. These are studied in Chapters XXXVI. to XLIV. on the assumption that costs of transport for factors of production, but not, so far as the last three of them are concerned, for commodities, are nil. In this discussion complications due to monopoly play a considerable part. In a many-commodity community the removal of the assumption about nil costs of transport for factors of production entails larger consequences than it does in a one-commodity community. Chapter XLVI. is devoted to these consequences; and Chapter XLVII. to some implications of State action. This completes our study of the thoroughgoing stationary state. In the final chapter a tracing is attempted of the intellectual journey that must be made from stationary states to arrive at actual life. Then follow fourteen appendices, in which portions of the subject-matter, that have been discussed in general terms in the text, are treated more precisely with the help of algebraic analysis.

CHAPTER II

THREE DEGREES OF STATIONARY STATES

§ 1. In every form of economic stationary state, even the least rigorous, the number, the age distribution, the sex distribution and the quality of the units that make up the population, the total amount of work that they do, and the total stock of capital equipment (as measured in the amount of work and waiting that goes to make it) must all be conceived as constant. Of course stationariness in this sense does not mean frozen fixity; individual drops composing the waterfall are continually in movement, though the waterfall itself remains. Wear and tear takes place, but it is always exactly offset by replacement. Moreover, the rates of wear and tear and replacement are constant; there are no jumps involving variations in the aggregate of work done by industries that make capital goods. These general conditions are, however, satisfied by three different arrangements, all of which may claim in a sense to be stationary states. First, the system of industry as a whole may be stationary, while the several industries that compose it are in movement. Secondly, every separate industry may be stationary, while the individual firms in it are in movement. Thirdly, individual firms, as well as individual industries, may be stationary. The characteristics of these three sorts of stationary state must be briefly described.

§ 2. In the least rigorous form, where only the system of industry in the aggregate is stationary, tastes may change in a compensating manner, now one thing becoming fashionable, now another; and technique may change, improvements in the method of manufacturing one thing being balanced

from time to time against disimprovements in the methods of making another. Thus it is not necessary that the distribution of the working population among different occupations shall be constant: nor yet that the quantity of capital instruments present in each occupation shall be constant. Shifts from one industry to another, from a factory for making cotton goods to a factory of equal real cost for making silk goods, and so on, are permitted. Replacements of men and machines need not be equal to displacements in particular occupations, though they must be equal to them on the whole.

§ 3. Provided that shifts in taste and technique take place with less than a certain critical speed, there need not, in a state of this kind, be any movement of persons or things from any one place or occupation to any other. The whole of the adjustment that is needed may—in theory—be accomplished through the replacement of men who die and machines that wear out in one occupation by new men and new machines entering other occupations. But, if comparative tastes and comparative technical efficiency in respect of different commodities vary too rapidly to allow of this, there must always be "excess capacity" of labour and equipment in some industries and deficiency of capacity in others. Thus suppose that the "season" occurs at one time of year in one part of the world and at another time in another part. Hotels not being capable of movement, there must always be, over the hotels of the two parts together, some excess rooms. In like manner some sorts of labour trained to meet summer needs cannot be transformed into sorts adapted to winter needs: and conversely. In general, rates of pay for agents, or factors, of production will be larger at any time in the industries that are then expanding than in those that are then contracting. The difference between the two sets of rates is at once the agency through which adjustment *tends* to be brought about and the evidence that this tendency is not completely actualised.

§ 4. In the second form of stationary state described in § 1 not merely the system of all industries, but each several industry, looked at as a whole, is stationary. This type of stationariness will be familiar to all readers of Marshall's

Principles. Just as in an unchanging group of persons individual men and women are born, come to maturity, grow old and die, so also in an industry that is stationary in this sense do the individual businesses that make it up. It is the unchanging forest; they are the trees that individually pass away. In every industry in a stationary state in this sense there is a certain constant price, which is required, other things being equal, to maintain any given aggregate annual flow of output. But, when this price rules and the aggregate annual flow is constant, some individual firms are "rising and increasing their output, others falling and diminishing theirs".[1] Thus the given price does not imply such and such a constant rate of output for each firm; a lower price would not imply a smaller rate of output for each, nor a higher price a larger one. Rather, a lower price would operate on the rate of aggregate flow by rendering the growth of the rising firms less rapid and the decay of the falling firms more rapid. At whatever level the price of the product stands, these rising and falling firms are not individually in equilibrium. But the industry as a whole is in equilibrium. It is behaving exactly as it would do if each individual firm were stationary, producing the amount that it is producing and under no stimulus either to expand or to contract; or alternatively, if the whole industry were divided up into a number of firms equal to the actual number and each producing the average output of all existing firms.

§ 5. There remains the third form of stationary state. In it not merely the system of industry and every individual industry stand constant, but also every individual business unit. This form contains all the attributes of the other two with a further attribute added. I shall speak of it as the *thorough-going* stationary state, and shall in this book focus attention on it. It will be found that a study of this simple and highly artificial model enables us to disentangle a number of complex interrelations that are fundamental in the real world.

[1] Marshall, *Principles of Economics*, p. 343.

CHAPTER III

A THOROUGH-GOING STATIONARY STATE IN RELATION TO STATIONARY CONDITIONS

§ 1. It is commonly assumed without discussion that, if in an economic community all the relevant governing conditions, people's attitude towards work and various sorts of commodities, their attitude to future satisfactions, the historical situation, the state of industrial technique and so on, are constant, the interplay of these conditions is bound ultimately to generate a stationary state. A long period, it is agreed, may elapse before this state is attained, but, so long as it is not attained, there is a disequilibrium, which the maximising principle is continually striving to remove, and eventually must succeed in, removing. The state finally attained may be one of nil action—an airless and waterless planet, in which no economic processes are taking place. But this, it is recognised, is still a form of a thorough-going stationary state. With stationary governing conditions, the common view is, *some* thorough-going stationary state—whether a nil state or some other state—must ultimately emerge. The purpose of the present chapter is to inquire how far this view is warranted.

§ 2. If the stocks of all factors of production were physically capable of indefinite enlargement, we might imagine them all expanding in equal proportions and the output of real income that results from them expanding in the same proportion. There would be nothing to prevent the demand prices for them in terms of product from remaining higher than the prices required to maintain the existing stocks, however large those stocks became. More generally, we can

imagine a situation in which the demand curves for stocks of the several factors of production stand above the supply curves, or as I shall prefer presently to say, the maintenance curves for them throughout their length. Demand and maintenance curves may not approach one another at all, or they may approach asymptotically without ever intersecting. In this situation stationary governing conditions would give rise to an eternally expanding economic universe, and not, even in the longest of long runs, to a stationary state. In actual life, however, an important factor of production, namely, land, or, more strictly, the natural resources which, for economists, the word land signifies, is limited in quantity. Though the stock of it in use may go on expanding for a long time, it cannot, in view of this limitation, do so indefinitely. This rules out the situation described above. If there is only one other factor besides land, this is obvious. The limitation of the stock of land entails the ultimate appearance, in respect of this factor, of diminishing returns; and so of equilibrium. If there are several other factors besides land, the situation is slightly more complex. A little reflection shows, however, that diminishing returns must emerge in respect of at least one of the other factors. Hence ultimately the stock of this factor must come into equilibrium and cease to expand. Thereupon diminishing returns, if they have not done so already, must emerge in respect of one at least of the remaining factors; and the stock of it too must eventually become fixed. This process will be repeated with all the factors, till ultimately the stocks of all of them have ceased to grow. In actual life, therefore, stationary governing conditions are not compatible with an eternally expanding economic universe. We may conclude that there necessarily exists some state, which, if it were attained, would be in equilibrium and stationary.

§ 3. It does not follow, however, that the equilibrium, which is thus attainable in thought, will be actually attained in fact, or even that closer and closer approximation to it will continually be made. The distinction, which physicists draw, between reversible and irreversible processes throws light on this matter. "Reversible processes include gravitation, mechani-

cal and electrical oscillations, sound waves and electromagnetic waves. Irreversible processes are found in the conduction of heat and electricity, radiation and all chemical reactions, in so far as their velocity is ascertainable." [1] The former sort of process is illustrated by an arrangement in which, in a U tube, a heavy liquid is set higher in one branch than in the other. When the higher standing liquid has fallen so that its level in both tubes is the same, it "does not come to rest, but moves beyond the equilibrium point on account of its inertia. . . . If loss of kinetic energy at the air surfaces and that due to friction at the walls of the tube could be eliminated, the liquid would oscillate upwards and downwards indefinitely over and under its position of equilibrium. Such a process is called reversible. Now in the case of heat the condition is quite other. The smaller the difference of temperature between the hot iron and the water, the slower is the transmission of heat from the one to the other. This means that there is always some difference of temperature, no matter how much time is allowed to elapse. There is no oscillation of heat, therefore, between the two bodies. The flow is always in one direction, and, therefore, represents an irreversible process." [2] This distinction is ordinarily not appreciated by laymen because, in the more familiar macroscopic problems of physics, friction is always present, so that the oscillating pendulum in fact does come to rest. But for my purpose it is very important.[3]

§ 4. Let us consider first the productive activity of factors of production when the stocks of all of them in existence are given and the governing conditions of demand and technique are stationary. Plainly there is a certain scale of activity and output which represents equilibrium. But suppose that, by some accident, in a particular year this equilibrium position is not attained, but the producers of some commodity are making more of it than in the ruling conditions of demand

[1] Max Planck, *Where is Science Going?* p. 189. [2] *Ibid.*

[3] The reader will understand that reversible and irreversible are technical terms of physics. It is not suggested that economic states, in which processes that are reversible, in the sense that a supply curve, drawn in one way for a forward movement, may have to be redrawn for a subsequent backward one, are necessarily states of oscillation.

is profitable. The price will then stand below the equilibrium price, and next year they will produce less. If economic processes were analogous to the irreversible processes of physics, they would reduce their output continually until the equilibrium amount *was* attained. But experience shows that in fact economic processes are not of this kind. Output next year will be cut below the equilibrium amount, and price will rise above the equilibrium price. Economists almost invariably assume without argument that there is present something analogous to physical friction, which will make the swings backward and forward round the equilibrium position gradually dwindle to nothing. But is this assumption warranted? Friction in macroscopic physics is a fact of experience. Whether anything analogous is at work to stop the movement of the economic pendulum cannot be decided *a priori*. It is a question of how people in fact behave. If they start one year by producing 10 per cent above the equilibrium amount, so far as *a priori* considerations go, the cut in production which disappointment causes may equally well carry their output next year more or less than that distance below the equilibrium amount. The swings of the economic pendulum are as likely to grow in amplitude as to diminish. What actually happens can only be ascertained by a study of the facts. Far too little work has been done on this subject to warrant dogmatism. The regular recurrence of the familiar 21-month pig-cycle necessarily raises doubts. The fact that an equilibrium position is *attainable*, which, if attained, would satisfy all parties, *may* ensure that that position will in fact be attained. On the other hand, there *may* come about a perpetual oscillation backwards and forwards round that position.

§ 5. The adjustment of the size of the stocks of fundamental factors of production to given governing conditions is much more akin to the irreversible processes of physics than the adjustment of the activity of existing stocks. For the total stocks of capital equipment and of labour are always large relatively to any possible annual increment; and, when disequilibrium exists, the process of adjustment is accomplished through the creation of a series of increments (or

decrements), the inducement to which becomes smaller and smaller the more nearly adjustment is attained. Thus, the conditions of demand being constant, it would seem that the stocks of the factors will approach towards and ultimately attain their equilibrium amounts, with no tendency to oscillate round these amounts. If increments (or decrements) emerged immediately after action had been taken to bring them about, this conclusion might be accepted with considerable confidence. But with capital there is some, and with labour there is a very long, interval between the creative act and the emergence of what it has created. If, for example, the stock of labour is too low to fit the rate of wages, and adjustment is approached by a raised birth-rate, the stock of labour will not be affected at all for some fifteen years.[1] For that period the stimulus to expansion will operate with a constant intensity. Thirty years from the initial date its full consequence is manifested. It may then be found that the stock has been increased beyond what adjustment requires. Oscillation about the equilibrium situation may be set up, instead of that situation being actually attained. Should this be so, stationary conditions will have failed to evoke a stationary state even in respect of stocks of factors of production.

§ 6. These considerations are of some practical, as well as theoretical, interest. Here, however, they will not be considered further. In the model to be studied it is postulated, as it were by fiat, that these oscillations do not occur. For that model constancy of all the relevant forces necessitates the establishment of a thorough-going stationary state.

[1] Cf. *post*, Chapter XXXI.

CHAPTER IV

THE CONDITIONS OF POSSIBILITY OF A THOROUGH-GOING STATIONARY STATE

§ 1. AT first sight it might be supposed that, so far as *a priori* considerations go, a stationary state might have any structure whatever; that to *any* actual state a freezing process could—in theory—be applied, in such wise that it would be turned into a stationary state and held so permanently. This is not so. A state cannot be frozen into stationariness unless at the time the freezing takes place it has a certain definite structure. The reason is that a stationary state must be stationary, not in one respect only, but in all respects; while it is only with certain structures that the several required kinds of stationariness are compatible with one another. This consideration has its most obvious application to population and to capital equipment. These two will be considered briefly in turn.

§ 2. In order to ensure stationariness it is clearly not enough that the birth-rate and the general death-rate shall be equal. This gives constancy of numbers, but it need not give constancy in age-distribution: and, if the age-distribution of a population varies, this will, in general, entail change both in its power as an instrument of production and in the direction of its choice among different objects of consumption. For example, a population with a large proportion of children and old persons is less productive than one that consists mainly of persons of working age: and, of course, children need different sorts of commodities from people in middle or later life. A stationary state, therefore, requires, not merely constancy in the total number of the population,

but also constancy of the numbers in every age group. This entails much more than mere equality of birth-rate and general death-rate. It entails also that the proportion of deaths in each age group is always such as to bring down by the next year the number of those who were living in any age group in the year before to the number who were then living in the next higher age group. But the death-rate of each group depends on conditions which must, for our purpose, be presumed to hold constant: otherwise the state is not stationary. It follows that the structure of the population in respect of age distribution must be such that the number at each age (n) exceeds the number at each age ($n + 1$) in a definite proportion, *i.e.* by the proportion of those at age n that are due to die in a year. Unless a population is initially of this structure, it *cannot* be frozen into stationariness. A population will necessarily be of this structure provided that, at the time when freezing is attempted, the birth-rate and the death-rate at every age have already remained constant for a period as long as the life of the oldest living person, but not otherwise. Even if it is not of this structure, it is still possible, of course, for it to be held henceforward for ever of a constant structure: but only on condition that the death-rates in the different age groups *change* from year to year: and a situation in which this happens cannot be called stationary.

§ 3. Analogous considerations hold for capital equipment. In a stationary state both the stock of equipment and the annual rate of consumption must be constant. This implies that the rate at which equipment wears out and has to be replaced is constant. For, if this condition is not satisfied, the stock of equipment can only be held stationary by varying amounts of income being withdrawn from consumption to meet depreciation in different years. Now, in order that the rate at which capital equipment wears out may be constant, the stock of it must be constituted in a specific manner. If there is only one sort of equipment, say, machines of a given kind, whose ages vary from n years to 1 year, the number of any one age must be the same as the number of any other age. It is impossible to freeze a state into stationariness at any stage during a period when either equipment is in process of

expanding or in process of contracting: for that implies that there are more young machines than old machines, or *vice versa*. Before freezing into stationariness can happen the age distribution of machines must *already* have been so adjusted that the numbers at each age are equal. If there are several different sorts of equipment, stationariness can obviously be introduced in a situation where initially there are the same number of units at each age for each sort of equipment, even though the highest age is different for different sorts of equipment. It can also be introduced, even though this condition is not satisfied, provided that one sort of inequality of age distribution among the units of one kind of equipment is compensated by a different sort among the units of another kind.[1]

§ 4. These matters have not much significance for the main purpose of our inquiry, and will not be developed further. Some reference to them, however, seemed, in the interest of logical coherence, to be needed. A more important requirement for the structure of a stationary state has to do with the *size*—we have here considered only the age-distribution—of the capital stock contained in it. This will be studied in Chapter X.

[1] Let the *values* of the total stock of equipment A, B, C . . . be v_a, v_b . . . and the proportion falling for renewal in a given year p_a, p_b. . . . It is then necessary that $p_a v_a + p_b v_b + \ldots$ shall be the same every year. With v_a, v_b . . . all constant, this condition is obviously satisfied if p_a, p_b . . . are all constant. But it can also be satisfied even though p_a, p_b are variable, provided that their respective variations compensate one another.

CHAPTER V

REAL INCOME AND FACTORS OF PRODUCTION

§ 1. In a stationary state factors of production are stocks, unchanging in amount, out of which emerges a continuing flow, also unchanging in amount, of real income. This in turn is distributed in unchanging proportions among the owners of the several factors. The amounts of the stocks, the rate of the flow, its detailed structure and the manner of its distribution are the dominant objective facts about which inquiry must centre. It is desirable, therefore, to provide ourselves with relevant descriptions or definitions, and so to delimit, at least in a rough way, the range of our investigation.

§ 2. Marshall, as is well known, defines the subject matter of economics as "those branches of conduct in which the strength of the motives chiefly concerned can be measured by a money price".[1] This statement needs, of course, to be properly interpreted. In different stages of economic development money plays, sometimes a larger, sometimes a smaller part. In a barter community it would play no part at all. Hence, if we were to follow Marshall literally, we should have to conclude that in such a community there would be no problems that could properly be called economic. Obviously he cannot have intended that. The difficulty is avoided if we say that economics has to do with those branches of conduct, or aspects of social life, in which *under present conditions* the strength of the motives chiefly concerned can be measured by a money price. Thus modified, Marshall's statement enables us to construct an inventory of those aspects of social

[1] *Principles*, p. 33.

life which at *any* stage of economic development belong to economic science. The inventory is not, of course, precise. There are a number of items on the margin between inclusion and exclusion. But this is inevitable. For Nature, here as elsewhere, has not drawn rigid lines. Accepting this general method of approach, we have to define real income in a manner accordant with it. This entails two steps; first, envisaging clearly the real *incomings* of goods and services of all sorts, regardless of whether or not they are connected with a money measure; and secondly, cutting out those that are not so connected.

§ 3. The first of these steps must be taken carefully. For it is easy to fall into confusion. Thus the real incomings of any period are often thought of as consisting of so many material things of various kinds *plus* such and such immaterial services. The material "things" contained in the income are, in this popular view, definite physical entities, which are first produced and then destroyed by use. But, of course, in fact, this is not so. The things that belong to real income, in their aspect as pieces of matter, are neither produced nor destroyed. For the broad purposes of everyday life nothing can be added to and nothing removed from the matter of the universe. Production is, in truth, a rearrangement of matter, and consumption a second rearrangement. The stuff of wheat after it has been consumed is, *qua* matter, precisely the same in amount as it was before. But it is matter in a form less well adapted to serve mankind. The "goods", which, at the end of the preceding section, we associated with services, are, in short, themselves nothing but services embodied in certain movements of juxtaposition or of severance, that have been given to particular pieces of matter. When we speak of them as physical entities, so much bread, boots, beer and so on, we are accurate enough for practical purposes, but we are not precisely accurate.

§ 4. There is also need for care in another respect. It is customary to speak of real incomings as accruing to people over some period of time. During this period services or goods, that have been produced in the earlier part of the period, may, before the end, have become embodied in other goods. When

this happens we must not count them twice over. If they are entered in our inventory in their original form, they must not also be entered in their embodied form: and conversely. Thus, when a loaf has been counted, and so the services of the baker embodied in it, those services must not be counted also as baker's services; nor must the fodder crops that are used for feeding pigs be reckoned in as well as the hog products, into which, in the same income period, they are converted. Following this line of thought further, we shall have to exclude such tools and machines as have been made in our period and used up in manufacturing finished goods: for the tools have, in effect, become embodied in these goods just as fully as, say, the coal that entered into their manufacture.

§ 5. This consideration leads up to a difficulty, which, when it is a question of defining real income in the shifting world of actual life, is very serious. It is necessary to disentangle the real incomings of any period from that part of the output of the period which merely serves to maintain the stock of capital intact by making good depreciation. But precisely to interpret the concept "maintaining capital intact" is not easy. It is not merely that, when some sorts of capital are diminished at the same time that others are increased, a balance has somehow to be struck. There are complicated questions about obsolescence due to changes of taste and new inventions. When, at the end of a war, the capital value of munition-making establishments collapses, what, if anything, is to be set off against this in reckoning up real incomings? Again, if an earthquake were to destroy half London, would the real incomings of the year in which this happened be depleted by the amount of the damage done? And, if this damage is to be treated as a capital loss, what of damage done by more ordinary types of accident? It is not impossible to answer questions of this kind in a fairly satisfactory way with the help of certain conventions.[1] For our present purpose, however, there is no need to attempt this. For in a stationary state the maintenance intact of capital—as of other agents of production—means simply its conservation in physically identical form. There are no new

[1] Compare *The Economics of Welfare*, Part I. chap. iv.

inventions and no changes of taste to cause obsolescence; no act of God or of the King's enemies to cause "abnormal" physical damage to capital. To maintain capital intact is simply to replace tools, plant and so on as they are worn out through use and through any decay that lapse of time associated with use may bring.[1] The part of gross output which is required for this and is, therefore, excluded from "real incomings" is thus definite and unambiguous.

§ 6. The reader will have observed that in the last section it is only to depreciation of capital that reference has been made. But natural resources—the stores of coal, minerals, oil and so on that Nature has laid up—are also liable to be depleted, which is equivalent to depreciation, by the processes that yield real income. Plainly here, too, some part of the gross output that comes into being is not real incomings. In so far as this is so our state is not in the strictest sense stationary; for, whatever we do, we cannot make good the "wear and tear" that is in this way inflicted on Nature's stores. Our concept, however, is not damaged seriously provided that we set aside in thought an amount of gross output adequate to provide some form of man-made capital equivalent in "value" to this wear and tear. In strict logic, it might seem, yet another slice of gross output should be set aside as the cost of rearing and training the new labour power that in every year or month is needed to replace the men who retire or die. If this were done, a considerable section of the goods that enter into ordinary consumption would have to be excluded from real incomings. Indeed, if the matter were pressed to the last analysis and food regarded as a mere means of restoring exhausted energies, far and away the greater part of gross output would be absorbed in this way, and real incomings would stand a tiny remnant. There can be no question here of a definition along these lines. The whole of what

[1] In fluctuating conditions it is desirable for some purposes to distinguish between the part of depreciation that results from use and the part that results from lapse of time, whether capital instruments are in use or are lying idle. But in a thorough-going stationary state there is no point in this. The instruments are always in use—for the ruling length of working day—and from our point of view both sorts of depreciation are the result of use, because, if it were not for use, the instruments would not exist and, therefore, could not decay.

human beings consume, whatever its relation to the maintenance of their energy or their numbers, belongs to real incomings. No allowance corresponding to that for making good wear and tear of capital or stored natural resources is permissible here. Real incomings are equal to gross output *minus* those two allowances, but *minus* those two only.

§ 7. The second step referred to in § 2 remains to be taken. To fit in with Marshall's use, we cut out from real income such part of this flow of real incomings as is provided in a non-commercial way—services rendered freely to one another by relatives and friends and services rendered to their owners by the durable consumers' goods that they possess and use —save only dwelling-houses. These last are included with the purpose of making real income correspond with income returnable for taxation, as understood by the Income Tax Commissioners. Plainly, paradoxes are involved in this. There is no distinction in logic between the services which a man obtains from a house that he owns and uses and those which he obtains from a motor car, or even a suit of clothes, that he owns and uses: and in neither case is the nature of the services rendered different when durable consumers' goods are retained in the owners' hands from what they are when these goods are hired out. In like manner it is paradoxical to make the sum of real incomes smaller if a man is married to his housekeeper than if he is paying her wages, or, if a woman, instead of cleaning a factory, is working equally hard and with the same efficiency in cleaning her own house. These paradoxes, however, so long as their existence is clearly recognised, need not lead to confusion in analysis; and it is a gain to have real income so demarcated that a measuring-rod in the form of money can be used in the study of it.

§ 8. Let us turn to factors of production. This term is a short phrase for factors of production of income. It is necessary, therefore, for coherence, that, when the definition of real income has been decided upon, the definitions of factors of production shall be made consistent with it. We have decided to confine the term real income to those "goods and services" that are normally bought with money, the only

important thing not so bought that is included being the services rendered by "houses and lands" that are occupied by their owners. Hence from factors of production are excluded objects of whatever kind, the services yielded by which do not fall into this more or less arbitrarily delimited category. It follows that some substantial elements of wealth—stocks of useful things—are not factors of production. Thus consumers' goods in the hands of final purchasers, however durable they are—yachts, motor cars, furniture—are not factors of production. These include only those entities "which yield income that is generally reckoned as such in common discourse; together with similar things in public ownership, such as government factories".[1]

§ 9. Factors of production thus conceived may, of course, be owned in a variety of ways. This is quite immaterial to the definition of them. A claim, such as a mortgage, constitutes a property right and yields income to the owner of it; but the income of the debtor is diminished by the precise amount of the mortgage interest. To a closed community as a whole these claims do not yield income, and must not be reckoned among factors of production. When, however, the members of one community have claims on the members of another, to the first community their claims yield real uncancelled income; and it is, therefore, necessary for consistency to reckon them, or, more properly, an appropriate share of the entities on which they are based, among the factors of production belonging to the first community.

§ 10. With this general understanding it is customary to divide the factors of production into three broad groups: income-yielding human agents; income-yielding gifts of Nature, grouped together under the name land; and income-yielding equipment, both material and immaterial (in the form of organisation), made by man and grouped together under the name capital. Under each of these broad heads there are, of course, innumerable sub-groups—an indefinite

[1] Cf. Marshall, *Principles*, p. 78. If, as is customary, we confine the term capital to goods that yield income, our decision to exclude the services rendered by durable consumers' goods in private hands from income entails excluding these goods themselves from capital.

number of kinds of labour, kinds of land, and kinds of equipment. For some purposes the distinction between particular sub-groups in a given main group is more important than the distinction between the main groups themselves.

§ 11. Since by definition all factors of production produce real income, it is self-evident that, in one sense, they co-operate together in doing this. But it is not implied in the definition that they co-operate together in the more natural sense of rendering one another mutual assistance. For all that the definition states, each several kind or sub-kind of factor of production might produce real income in isolation from, and quite independently of, all the rest. To some extent this actually happens. But most elements in a community's real income are produced by the joint operation of two or more kinds or sub-kinds of factors, in such manner that the different factors or sub-factors acting in combination yield more than equal quantities of them acting separately would do. In some cases factors, which, when combined with others, are highly productive, by themselves could produce nothing at all. This is not true of human agents, or of land in the narrow sense, or of capital embodied in land in the form of drainage and so on; but it is true of all kinds of capital embodied in machines. It is only when tended and directed that this factor of production wakes to life.

§ 12. A certain awkwardness, it may be noted, arises from the fact that labour sometimes co-operates with, in the sense of assisting in work, durable objects that are not reckoned as capital. The chauffeurs that look after the motor cars of private persons and the crew of a millionaire's yacht are in this position. That the services of these men should be part of the national income, while the services of the motor cars and the yacht are not, appears especially paradoxical—more so than the exclusion from income of the services of durable non-capital goods that function without the help of labour. For inquiries about the division of income between capital and labour it would seem proper either to exclude the income of labour employed in such ways or to add something to total income as here defined, so as to allow for the part played by the motor cars and yachts.

§ 13. It will be understood, of course, that in the preceding sections we have been concerned to construct convenient definitions, not to carry out an analysis. If that had been our object, we should not have been entitled to set up "capital" as a co-ordinate factor along with labour and land. For capital instruments are themselves produced by labour, existing capital instruments, waiting and, it may be, since our stationary state may have arisen out of one which was not stationary, uncertainty-bearing. If the analysis were pushed back far enough in time, capital instruments would all be found to originate ultimately in labour, land, waiting, and again, it may be, uncertainty-bearing, and thus to constitute a derivative, not an ultimate, factor of production. Moreover, had we been concerned to follow this line of thought, we should have needed to analyse further the stock of labour as it stands at any moment: for this is itself partly the product of waiting in the past. It is more convenient, however, to regard such elements as "waiting" and "uncertainty-bearing" as *sources* of factors of production than as factors themselves: the factors themselves being defined in the manner adopted here.

CHAPTER VI

THE SIZE OF REAL INCOMES

§ 1. If real income were made up of a single sort of commodity only, its size would always be measured by the number of units of that commodity contained in it. A comparison of the sizes of the real incomes in two or more stationary states would then present no difficulty. With real incomes made up of many different sorts of commodities this is not so, and, as a consequence, awkward issues have to be faced.

§ 2. If of two real incomes one contains more units than the other of some items and not less units of any item, everybody would agree that it is larger than the other. There is no doubt or ambiguity about that; though, unless it contains the same *proportionate* excess of every sort of commodity, there is no definite percentage *by which* it is larger than the other. If income A contains more units of some commodities and also less units of any commodity than income B, income A cannot in a strict physical sense be said to be either larger or smaller than income B. For the different commodities are not commensurable, and there are no physical grounds for assigning to them any one set of relative weights rather than any other.

§ 3. It may perhaps be thought that the difficulty can be overcome by comparing real incomes, not in themselves, but in respect of their values. It is, of course, always possible, with a pricing system, to value each of two real incomes in terms of any commodity that we choose, and to set the values so reached over against one another. This is frequently done in terms of money. Unfortunately, however, the two valua-

tions will, in general, be related to one another in different ways according to what commodity is taken as the measure of value. Thus, suppose that we have two incomes each comprising items of three sorts—A, B, C; that in the first income the quantities of these items are a, b, c, with money prices p_a, p_b, p_c; and in the second α, β, γ, with money prices π_α, π_β, π_γ. The money value of the first income divided by that of the second is then $\frac{ap_a + bp_b + cp_c}{\alpha\pi_\alpha + \beta\pi_\beta + \gamma\pi_\gamma}$. Call this m. The value of the first income divided by that of the second in terms of commodity A is $\frac{\pi_\alpha}{p_a}m$; in terms of commodity B, $\frac{\pi_\beta}{p_b}m$; in terms of commodity C, $\frac{\pi_\gamma}{p_c}m$. These quantities are obviously, in general, different. There is nothing to prevent one of them being greater, while another is less, than unity. Thus the result of comparisons depends on the choice we make of the commodity in terms of which valuations are to be made; and this is purely arbitrary. Nothing useful, therefore, can be accomplished on this plan.

§ 4. A more hopeful method of approach is to *define* greater or less real incomes by reference to the satisfactions which they yield, or, what is not quite the same thing, the desires that they satisfy. If both of two incomes are enjoyed by groups of people equal in number with precisely similar tastes, we may say, if we will, that that income is the greater the aggregate satisfaction from which is the greater. This is a clear-cut concept. Starting with it, we may attempt to derive numerical indices embodying quantities and prices, designed, as between different real incomes, to be larger or smaller according as the associated aggregate satisfactions are larger or smaller. This plan is developed in Part I. chap. vi. of *The Economics of Welfare*. For the purposes of welfare economics there is much to be said for it; though it is impossible either to avoid the introduction of arbitrary elements or to devise an index that is more than *likely* to point in the right direction. For the positive inquiry of this book that way of approach is, however, clearly not appropriate.

§ 5. None the less we need not, I think, refuse altogether to go beyond the statement of § 2, that income A is larger than income B if it contains more of some commodities and not less of any commodity. Over and above this two further statements—they are, in effect, definitions—may properly be made. First, if the state of productive technique in respect of every industry is the same in two periods, but in one period more of some factors and not less of any factor is at work than in the other, the real income of the first period is larger than that of the second. If everybody's tastes are the same in the two periods, this says nothing more than has been said already; for in these conditions the quantity of each sort of commodity contained in real income must be larger in the first period. But our new statement enables us to add that, even though tastes are different, and, as a consequence, less of some commodity is being produced in the first period than in the second, real income is larger in the first period. Secondly, if the quantity of every several factor at work is the same in the two periods, but in one of them productive technique is better in some industries and not worse in any industry than it is in the other, and if, all other things being the same, the distribution of productive resources among commodities differs solely on account of differences in technique, the real income of the first period is the larger. This covers the case where an improvement in an industry of elastic demand draws factors of production into that industry from elsewhere, and so leads to a reduction in the output of other industries.

CHAPTER VII

DETERMINATE AND INDETERMINATE SITUATIONS

§ 1. In the course of our inquiry it will be necessary more than once to raise the question whether such and such a situation is economically determinate or indeterminate. It is well, therefore, at the outset to clear our ideas as to what precisely this question means. When *all* the relevant antecedents are given, it has until quite recently been a postulate of science that the resultant situation *must* be uniquely determined. In the last decade some physicists have, indeed, come to believe that in the universe of the very small this postulate does not fit the facts; that within the atom there is a principle of indeterminateness; that the behaviour of individual electrons is not merely the result of causes that are unknown, but is "uncaused" and the sport of "objective chance". But we have not to do with these high matters. For our field of study, we accept, as we are bound to do, the postulate of unique causal determinism. When questions about economic determinateness are raised there is no intention of casting doubt on that. An economic situation can never be indeterminate in an absolute sense. It can only be indeterminate relatively to given data.

§ 2. An awkwardness arises here on account of the way in which the terms determinate and indeterminate are commonly used in mathematics. For that science a problem is determinate when the elements of which we are able to take account can be set out in a number of independent equations equal to the number of the unknowns. It is indeterminate if the number of independent equations is less

than the number of unknowns.[1] With these definitions, if every equation necessarily possessed only one solution, mathematical determinateness would imply economic determinateness, and mathematical indeterminateness economic indeterminateness. In fact, however, a system of n independent equations containing n unknowns, even though n be unity, may possess a large number of alternative solutions. It follows that, though mathematical indeterminateness implies economic indeterminateness, mathematical determinateness does not imply economic determinateness. For that there may be needed further data, which will allow choice among the alternative solutions to be made. If our data are that each party in a given environment is seeking to better himself by a bargain, and that the terms of contract are agreed to by the free consent of both parties, the situation is economically determinate relatively to these data, if and only if the data suffice to ensure that a single unique rate of exchange will inevitably be arrived at. Equality between the number of independent equations and the number of unknowns does not necessarily suffice for this.

§ 3. In the matter then of economic determinateness and indeterminateness a stationary state and a non-stationary state are on all fours. Relatively to all the data both alike are economically determinate. Relatively to the available data either may be economically indeterminate. If either of them is economically determinate, this implies that the data can be set out in a number of independent equations equal to the number of unknowns, and also, when there are alternative solutions, that some further data are available to discriminate one from the rest. A stationary state, which is determinate relatively to the available data, differs from a non-stationary state, which is similarly determinate, merely in the fact that

[1] If there are more independent equations than there are unknowns, the problem is said to be "too determinate". More conditions are given than are necessary to solve the problem. If the conditions are all truly independent, they cannot, except by a miraculous accident, all be mutually compatible. Hence the problem is a nonsense problem. We are given, in effect, as data both that a particular table is round and also that it is square. To a problem of this kind there is obviously no solution. The implication is that the conditions set out in the problem cannot in fact coexist in the real world, so that the assumptions made in stating it must be incorrect.

in it the solution of the equations is the same for all values of that unknown which measures passage of time. Whether either state is in truth economically determinate in respect of the available data cannot, of course, be decided *a priori*, but only on a study of the facts.

CHAPTER VIII

THE ECONOMIC LIFE OF ROBINSON CRUSOE IN A STATIONARY STATE WHERE EXISTING FIXED CAPITAL LASTS FOR EVER AND THERE IS NO WORKING OR LIQUID CAPITAL

§ 1. For anyone whose interest in Economics is a practical one, to be asked to contemplate, not merely the fiction of a stationary state, but an isolated Robinson Crusoe in this state, cannot fail to be annoying. He is not interested in imaginary individuals living solitarily upon islands, but in the world as it actually is. Robinson Crusoe economics is, he will complain, an academic plaything without practical significance. It is as though he were invited to contemplate an elephant whose weight may be neglected or an Atlantic liner with neither length nor breadth: everything that gives the subject significance or reality is deliberately left out of account. This objection, natural as it is, is not well founded. The defence for Robinson is the same as that for the stationary state itself. He is a first step—a means to an end. No one objects to a writer on dynamics beginning his exposition with an account of imaginary bodies falling in a perfect vacuum. Nobody should object to an economist's resting for a time on Robinson's celebrated island.

§ 2. Robinson, then, is a man who is perpetually engaged, under the sway of the principle of maximum satisfaction, in balancing desires against aversions from the means by which desires are satisfied. He desires various sorts of things, food, clothing and so on. In order to obtain these things he must work. On what plan does he decide the amount of work which he will perform and the manner of its distribution? Since he is living in a stationary state, the hindering effect of

miscalculations and of those various sorts of friction, which in real life play an important part, are non-existent; so that what at any moment Robinson desires to do, that he in fact does do. In this chapter attention is confined to a highly simplified case, in which it is supposed (1) that existing fixed capital lasts for ever; (2) that all services and goods emerge immediately at the moment when work is applied to produce them, so that there cannot be any working capital; and (3) that they are all instantly perishable and incapable of being held or stored, so that there cannot be any liquid capital.

§ 3. A preliminary word of explanation is here desirable. This has to do with the definition of work. Robinson's work is *not* a one-dimensional entity measurable simply in terms of the time spent upon it. The units appropriate for measuring it are efficiency units, in which quality and intensity, and not duration only, are brought into account. An efficiency unit is a small entity containing an arbitrarily chosen amount of quality, intensity and duration. A piece of work contains 10 units if it can be divided up into ten pieces, the addition of each of which, *singly without the others*, to the existing environment would increase total output precisely as much as the addition of the above small entity would do. In saying this, it must be observed, I am not defining a unit of work as that quantity which produces a unit of output. My definition is genuinely objective, and not of a kind to make the proposition that constant returns prevail a tautology.

§ 4. With this understanding let us begin by supposing that there is only one, and not several, sorts of thing that Robinson's work yields, so that all he has to decide is how much work he will do, without troubling about the way in which it is distributed over different tasks. At first blush we may incline to say that in any week or day he will work in such measure that his desire for the total product of his work exactly offsets his aversion from the work itself. But that is not the kind of balancing by which his conduct is regulated. If he were under compulsion to choose between working, say, fifty hours in a week and not working at all, he would, indeed, work or not work according as his aversion from those fifty hours was less or more than his desire for their total product.

If the aversion were the stronger he would not work: if the desire were the stronger he would work for fifty hours. We should be able to say this and nothing further. But, in fact, Robinson is not tied down to the narrow choice between fifty hours' work and no work at all. Work for him, and, in like manner, the product of work, is divisible into small units. For the purposes of this chapter we postulate that the process of division can be carried, without cost and without alteration in the character, either of the work or of its product, to the extreme limit of smallness. Robinson is free—to borrow Marshall's illustration—to go on picking blackberries, or whatever his work may be, for just so long as he wishes. Plainly then he will stop, not when his aversion from the whole time spent in picking balances his desire for the yield of the whole time, but when his aversion from another second's picking balances his desire for another second's product of blackberries. Work, in short, will be carried so far that aversion from work and desire for the product of it balance one another, not in the aggregate, but at the margin: in other words, till he is doing such quantity of work that his aversion from the marginal unit of work is equal to his desire for the product of the marginal unit, or, more briefly, for the marginal product. This marginal product is, of course, conceived as net, after allowance has been made, on the lines of Chapter V., §§ 5-6, for any damage suffered by any capital instrument that Robinson may be using or by stored natural resources.

§ 5. This principle, it is easy to see, is of wide generality. It is equally valid whether Robinson's aversion from work, his desire for its product and his productive efficiency are independent of one another or are interconnected. It is customary, no doubt, to assume independence among these things. But in fact, the longer a man stokes a furnace, the more keenly will he desire the hot bath with which it provides him; while access to a hot bath will render him at once more willing to resume the task of stoking and more efficient in performing it. When desire for product, aversion from work and productive capacity are independent of one another, the mathematical formulae, in which the facts can be expressed, are simpler than those needed when they are

not independent; but the fundamental principle of balance at the margin is in either case the same.

§ 6. In the foregoing discussion an important element has been concealed: namely, the relation between the quantity of work that Robinson performs and the size of the marginal product. So far we have been content to say that his desire for this marginal product, whatever it is, must balance his aversion from the work devoted to making it. We have now to bring into the picture what is sometimes called the productivity function of Robinson's work. Even when all the environing circumstances are given, the size of the marginal product, *i.e.* the difference that would be made to Robinson's total output by withdrawing, in the least injurious manner,[1] one unit of his work, may be different according to the quantity of work that he is performing; and it *must* be different according to the degree of help which Nature, through good or bad soil, climate and so on, affords to his labour. In determining how much work he will do the productivity function of Robinson's work, therefore, plays an important part. His desire for the marginal product of any given quantity of work is related to the relevant productivity function in a double way. First, the size of the marginal product of any given quantity of work—say 100 units—depends on it. Secondly, the total amount of product that Robinson obtains from the whole 100 units, and, therefore, the keenness of his desire for the product of the marginal unit, *i.e.* for the marginal product, whatever its size may be, is affected by it. The nature of the relation involved is readily expressed in the symbolism of the differential calculus. Readers who are unable to handle that very useful tool must do the best they can with ordinary language. But it is idle to pretend that they are not handicapped.

[1] It will be understood that the marginal unit of work is not a specific unit, but the difference between two adjacent numbers of units. Moreover, if the several specific units are employed in varying conditions, *e.g.* one on good land and another on bad, the product of the marginal unit is measured by the effect on total product, not of withdrawing a unit engaged on the good land or on the bad land, and doing nothing else, but of withdrawing this unit and redistributing bits of other units in such a way that the marginal yield of work is once more, as it was originally, the same on all sorts of land.

§ 7. So far we have provisionally supposed that Robinson's work is devoted to producing only a single sort of thing. In fact, of course, it is spread over a great number. Some of these things are produced singly, others in a joint group by a common process. In any case, Robinson has now to determine, not only how much work he will do in the aggregate, but also how he will distribute his work among different occupations and processes. Let us assume, for simplicity, that an hour's work means the same thing to him to whatever purpose it is devoted. Clearly then, in accordance with the reasoning of the preceding section, he will push his work in each several occupation up to the point at which his desire for the marginal product of the work done there and his aversion from the marginal unit of that work balance one another. But now a new element comes in. If Robinson were in fact producing only one sort of thing, his aversion from the marginal unit of work devoted to it would depend only—apart from the type of connection mentioned in § 5—on his general attitude towards work in conjunction with the quantity of work that he is doing on that thing. Since, however, he is in fact producing a number of different sorts of things and doing work at all of them, aversion from providing the marginal unit of work in respect of any one of them depends, not on the amount of work devoted to that one alone, but on the amount devoted to all of them collectively. His marginal aversion from work must thus be the same in all occupations: whence it follows, as a corollary, that he so spreads his work over different things that his desire for its marginal product in any one occupation is equal to his desire for its marginal product in any other.[1] Thus the balancing process does not take place separately at each margin. It operates through a system, all the parts of which are connected. This is so equally whether Robinson's desires for different things are independent of one another or are interdependent. For certain things his desire will be keener, the

[1] This need not be so if Robinson's attitude towards work is of a more complex kind, *e.g.* if his aversion from the rth unit of work in a given occupation depends partly on the size of r and partly on the quantity of work that he is doing in all occupations together.

worse position he is in for obtaining certain other things. He will be more anxious, for instance, to receive a given rth unit of beef if mutton is scarce than if it is abundant. *Per contra*, for certain things his desire will be less keen, the worse position he is in for obtaining certain other things. He will not, for instance, care so much for a given rth unit of tea if sugar is scarce as he will do if it is abundant.[1] Alike when these interrelations do and when they do not exist the process of balance at the margin governs at once the total amount of Robinson's work and the proportions in which he distributes it along different lines of activity.

§ 8. The whole complex of elements that come into balance can, if it is so wished, be represented with complete generality. The picture is, however, easier to grasp if it is simplified. Let us suppose, then, that Robinson's productivity functions for different commodities are independent of one another; that his desire attitudes for the several products are independent of one another; that these desire attitudes are independent of the amount of work that he does; and that his aversion from work is everywhere independent, alike of the way in which it is distributed and of the amount and character of his consumption. In these conditions the amount and distribution of his work per week or year are controlled by the interplay of three sets of factors: (1) his desire attitudes towards each several product; (2) the productivity function of his work in each several occupation; and (3) his aversion attitude from work. No one of these elements is a single thing or capable of being expressed by a single number. For Robinson's desire for the 100th unit of any product is sure to be different from his desire for the 10th; the efficiency of the 100th unit of his work in producing commodities may be different from the efficiency of the 10th; while his aversion from the 100th unit of work is sure to be different from his aversion from the 10th. The situation is thus somewhat complicated. Nevertheless, when the elements described above are given, it is easy to see that they are sufficient, in conjunction with our principle of balance, to determine, in the mathematical sense of that word, how much work per

[1] Cf. *post*, Appendix II.

week or month Robinson will elect to perform in each occupation. For the number of unknowns and the number of independent equations available are obviously equal.

§ 9. This conclusion by itself does not enable us to say whether the maximum principle avails to determine Robinson's conduct uniquely in a causal sense, or merely delimits for that conduct a certain set of alternatives, the choice among which must be settled by other influences. *A priori* there is no reason to expect that a single solution only will satisfy the conditions; there may be many equilibrium states of the system, not one only. Closer investigation, indeed, shows that, when multiple positions of equilibrium exist, some of them are, in general, unstable positions, such that the very slightest disturbance—as with an egg balanced on its end—would cause the system to run down to some other situation. These unstable positions are not capable of being held in real life and may be disregarded.[1] In general, however, if a system of equations has more than one solution altogether, it will have more than one solution that represents a stable position. Hence this consideration does not destroy the *a priori* likelihood that several *effective* alternative solutions will be available. Whether in fact there are several or only one such solution depends upon the detailed nature of the governing conditions. We may reasonably premise that in our hypothetical stationary state

[1] In Marshall's treatment all positions of equilibrium are regarded as stable except those to which, after a very small disturbance, the system will not tend to return. It is important, however, not to forget that there are some states of equilibrium, stable in this sense, which, while they can sustain disturbances up to a certain magnitude, are not stable against larger disturbances. Professor Irving Fisher instances a twig that will straighten itself again after some measure of bending, but with greater bending will break, or, a ship, which, if it rolls beyond a certain critical degree, will not tend to right itself but will capsize. (*Econometrika*, October 1933, p. 399). Strictly, no doubt, in a stationary state these equilibria are on all fours with those that enjoy an absolute stability against disturbances no matter how large. We may, none the less, permit ourselves the observation that in real life the important distinction is, not that between stable and unstable equilibria in Marshall's sense, but that between equilibria which are respectively stable and unstable as against disturbances of the order of magnitude to which they are likely in ordinary circumstances to be exposed. The phrase "ordinary" circumstances is, of course, ambiguous. A precise definition could, however, be framed to fit the conditions of any particular problem, provided, of course, that adequate data were available.

Robinson's desire for an extra unit of anything diminishes as the quantity of it that he has increases, and that his aversion from an extra unit of work at anything increases as the quantity of work that he is doing increases. If we could also premise that the marginal productivity of his work at each sort of product diminishes as the amount of work he does at it increases, or even that it does not increase fast enough to cause the desiredness of the fruit of the marginal unit of work to increase, one solution only would be possible. But we are not in fact entitled to premise either of these things. It follows that there may—not necessarily must—be several alternative arrangements, any one of which is compatible with the governing conditions set out in the equations, and decision between which depends on extraneous circumstances —if we will, upon "the historical situation".

CHAPTER IX

A COMPARISON OF DIFFERENT STATIONARY STATES IN CRUSOELAND WHERE EXISTING FIXED CAPITAL LASTS FOR EVER AND THERE IS NO WORKING OR LIQUID CAPITAL [1]

§ 1. LEAVING aside the complication noticed at the end of the last chapter and postulating that we have to do with conditions such that multiple solutions are excluded, I shall now, to complete the picture, compare in a general way the scale and distribution of Robinson's activity in different stationary states. To this end let us inquire in what way the amount and distribution of his work are related to different conditions of (1) his scheme of desire attitudes towards different commodities, (2) the productivity functions of his work on different commodities and (3) the function that describes his aversion from work. This inquiry is most easily conducted if we suppose ourselves to start with one state of the relevant elements, to which Robinson's conduct is fully adjusted, imagine in turn changes in the state first of one relevant element and then of another, and ask in each case how the amount and distribution of Robinson's work, *when fully adjusted to the new situation*, will differ from what it was initially. It is assumed, as in § 8 of the last chapter, that Robinson's productivity functions for different commodities are independent of one another, as are also his desire attitudes; that these desire attitudes are independent of the amount of his work; that his productive capacities are independent of his consumption; and that his aversion from work is (1) related only to its aggregate amount without regard to its

[1] Cf. *post*, Appendix III.

distribution and (2) independent of his consumption. It is further assumed that Robinson's productivity functions are all of such a character that any given proportionate increase in quantity of work entails an equal or a smaller—not a larger—proportionate increase in output: *i.e.* that constant or diminishing, not increasing, returns prevail. This is a reasonable assumption: for, as will appear presently, with a single Robinson co-operation and specialisation of jobs, which in actual life sometimes promote increasing returns, are, from the nature of the case, absent. Finally, though we cannot describe the nature of Robinson's attitude of desire for any commodity or his aversion from work in detail, experience suggests that, other things being equal, his desire for an additional unit of any commodity is likely to be less, the more of it he is already receiving, and his aversion from additional units of work to be greater the more work he is already doing. We assume that these suggestions correctly portray the facts.

§ 2. Owing to the fact that the relevant elements are not single quantities of desire, aversion and productivity, but lists, or schedules, of quantities, the changes to which they are liable are complex in character. Of none of them are we able to say that it can only alter by increasing or decreasing, by rising or falling. This is most easily seen if we envisage, as we are entitled to do, any one of Robinson's several desire functions or productivity functions as represented by a curve. Thus let us imagine a curve D_1, so drawn that perpendiculars suspended from successive points on it measure Robinson's desire for successive units of wheat per week or month at the first of the two times we are comparing, and a second curve D_2 drawn on the same plan in respect of the second of the two times. It is obvious that D_2 may be related to D_1 in an infinite number of ways. It may lie wholly above it or wholly below it; or it may intersect it once or twice or any number of times. The same thing is equally true of Robinson's aversion schedule from work and of each one of his productivity schedules. It is out of the question to describe the effect on his conduct of all the innumerable sorts of change in any, much less in all of the relevant elements,

that are possible. We must perforce confine ourselves to a few main types.

§ 3. To this end we rule out changes that would be represented by the substitution for any original curve of desire or productivity or aversion from work of a new curve that intersects with it. For it is unlikely in practice that, if Robinson comes to desire the 10th unit of anything more than he used to do, he will at the same time come to desire the 100th, or any other, unit less than he used to do; and the same thing is true of his powers of production in any field and of his aversion from work. We have to do only with shifts from one curve to another curve that lies throughout its length either above it or below it. Robinson's desire attitude for anything has intensified or the reverse in respect of every unit of it; his productivity in any field has increased (or the reverse) in respect of every unit; his aversion from work has increased (or the reverse) in respect of every unit. On this basis, with the assumptions of § 1, we are able to reach certain broad conclusions. These are set out in ordinary language in the sections that follow, and demonstrated by algebraic analysis in Appendix III.

§ 4. Before they are set out it will be well to introduce and define a concept, which, throughout this book, will play a very important part. This concept is that of *elasticity*. The elasticity of marginal desire for a commodity, in respect of any quantity of it, is measured by the proportionate change in marginal desire divided into the associated proportionate change in the quantity of commodity, when the size of the smaller of these changes is very small. In like manner, the elasticity of marginal productivity, in respect of any quantity of work, is the (very small) proportionate change in marginal productivity divided into the associated proportionate change in quantity of work; and the elasticity of marginal aversion from work is the (very small) proportionate change in marginal aversion divided into the associated proportionate change in quantity of work. At a later stage a number of other elasticities will come into the picture, all defined on the same general plan. For the purposes of the present chapter the three that have been distinguished above are the only ones

required. In the economics of real life there is, for all of them, the serious difficulty that the relation between the two proportionate changes that they connect is liable to be different according to the length of time for which either change has endured, so that there are a number of different elasticities relevant respectively to short, intermediate and long period problems. The economics of stationary states, however, are free of this difficulty. The only relevant elasticities are those proper to long periods, *i.e.* those which measure the relations between our two proportionate changes when full adjustment to the new situation has been made.

§ 5. Let us begin by supposing that Robinson is producing only a single sort of commodity, and consider the consequences of a rise in his desire for the commodity, a fall in his aversion from work and an improvement in his productivity. It is evident—on our assumption of constant or diminishing returns—that either of the first two sorts of change must lead to his both doing more work and obtaining more of the commodity; and that the increase, both in quantity of work and quantity of commodity, will be larger, alike for a rise in desire for the commodity and for a fall in aversion from work, the greater is his elasticity of marginal desire for the commodity, the greater is his elasticity of marginal aversion from work, and the less strongly diminishing returns operate. Attention will, therefore, be concentrated on the third case, that of an improvement in productivity. It is easy to see in a general way that, if Robinson's marginal aversion from work is highly elastic and his marginal desire for the commodity highly inelastic, since in these conditions he does not much care for extra product but gains a good deal by cutting down work, he is likely to do less work after the improvement than before; and in converse conditions to do more work after the improvement. There must, therefore, always be some critical intermediate situation in which the amount of work that he does is not changed. This critical situation cannot, however, be determined if we merely know that the improvement conforms to the conditions set out in § 3. For under those conditions an infinite number of different types of improvement

are comprehended. To obtain a precise result we must specify precisely what type of improvement has taken place. It is natural to choose for study the simplest type, in which the marginal (and total) products of all quantities of work are affected in the same proportions. Let us call this type of improvement an *equiproportional* improvement. After this type of improvement it can be shown that Robinson will do more or less work than before according as his elasticity of marginal desire for the product, in respect of the amount originally produced, is numerically greater or less than unity. In either case the quantity of work that he does will increase more (in the sense which, if it is decreasing, includes will decrease less) the larger numerically is that elasticity. The increase, if it is an increase, will be larger, and the decrease, if a decrease, will likewise be larger, the more elastic are Robinson's marginal aversion from work and his marginal desire for product, and the less strongly, if at all, diminishing return operates.

§ 6. A caution must be added here against a possible false inference. It may be thought that, in conditions such that Robinson does less work after the improvement, he *may* also secure less total product. That this should happen is, indeed, common sense tells us, very unlikely; for that a man should in fact get less stuff in consequence of becoming able to get more is a paradox. But to prove that this paradox *cannot* be realised is not easy by purely verbal argument. In truth, however, with any sort of improvement, as defined in § 3, not merely with equiproportional improvements, a cut in quantity of product is impossible. Robinson *must* secure more product. This is proved in Appendix III., Problem 5.

§ 7. Let us next allow Robinson to devote his efforts to producing, not one commodity only, but a number of different commodities. There will be considered in turn the consequences of shifts in (1) his desire attitude in respect of one of them, (2) his productivity in respect of one of them, (3) his productivity in respect of all of them and (4) his aversion attitude towards work. The easiest way of tackling these issues is to study them first on the assumption that conditions of constant return rule everywhere, and, thereafter, to

indicate the modifications in our conclusions that are necessary when this assumption is not warranted.

§ 8. First consider an intensification of Robinson's desire for some commodity, say wheat. Clearly this carries with it an increase in the quantity of work that he performs in producing wheat, and so also in the quantity of wheat that he receives. The quantity of work that he performs in respect of other commodities and the quantity of them that he receives will be diminished; but the aggregate quantity of work that he does will be larger than before. The extent of the increase in aggregate quantity of work will be larger, the more elastic is his marginal desire for the favoured product and the more elastic is his marginal aversion from work. This is fairly evident to common sense; for in the extreme case of nil elasticity of marginal aversion from work there obviously cannot be any increase. It will also be larger the less elastic is his marginal desire for each of the other commodities; because, if his marginal desire for any of these is elastic, he will obtain a good deal of the extra work directed to the favoured commodity by cutting down work on that one, whereas, in respect of commodities for which his marginal desire is inelastic, this resource is not so readily available.

§ 9. Secondly, consider an improvement in Robinson's capacity for producing some one commodity. The consequences of this are less obvious. In the new situation, *if* he does more work than before in respect of the commodity affected by the improvement, he will do less work in respect of other commodities and more in the aggregate; and conversely. But, whether he will in fact do more or less work in respect of the particular commodity depends on the character of the improvement that has taken place. Let us, as before, suppose that it is an equiproportional improvement. This kind of improvement entails more or less work in respect of the favoured commodity—and so also in the aggregate—according as his marginal desire for that commodity has, over the relevant range, an elasticity greater or less than unity. In either case, if his productivity function rises in a given proportion, the aggregate quantity of work that he does will increase more (in the sense which includes, if it decreases,

will decrease less) the more elastic is Robinson's desire for the commodity in respect of which the improvement has taken place. If this elasticity is numerically greater than unity, it will also increase more (i) the more elastic is his marginal aversion from work, and (ii) the less elastic are his marginal desires for each of the other commodities. If the elasticity of Robinson's marginal desire for the commodity directly affected has an elasticity less than unity, the aggregate quantity of work that he does will decrease more, the more fully these two conditions are satisfied.

§ 10. In the preceding section we have shown that in certain conditions an improvement in Robinson's productivity in respect of some one commodity will cause less of his work than before to be devoted to producing that commodity. It may be thought that, whether this happens or not, it is impossible for him, as a consequence of the improvement, to obtain a smaller quantity of the commodity primarily affected, provided that, as we have been assuming, his desire function for it is independent of his desire functions for all other commodities. Sir Robert Giffen's well-known paradox, that an improvement in the methods of manufacture, leading to a fall in the price of bread, *may* bring about a decrease in the quantity of it that people buy, may be supposed to depend upon the fact that different sorts of food are rivals to one another, *i.e.* that the desire functions for them are interrelated—a state of things which is here expressly excluded. This is not so. As is shown in Appendix III., Problem 6, the paradox may appear without that condition being satisfied.

§ 11. Thirdly, consider a uniform proportionate upward movement in the whole body of Robinson's several productivity functions. The proportion of his work devoted to commodities of more elastic marginal desire—in respect of commodities the desires for which are independent—will rise relatively to that devoted to commodities of less elastic marginal desire. Thus a general improvement of productivity causes a larger proportion of all the work done to be devoted to luxuries and a smaller proportion to necessaries. If the elasticities, over the relevant range, of all Robinson's different

desire functions were equal, an equal proportionate improvement in productivity all round would, of course,—since we are postulating constant returns,—leave the proportion of work that he devotes to different commodities unchanged.

§ 12. Turn now to a shift in Robinson's aversion attitude towards work. We are assuming, it will be remembered, that work in any one occupation is the same to Robinson as work in any other, so that his aversion from the marginal unit of work depends solely on the total quantity of it. It is then evident that a decrease in his aversion from work—a fall in his aversion schedule—will lead to his doing an increased amount of work in every occupation—unless there is some occupation in which either additional work would yield no product or additional product would encounter no desire. The amount of work that he does in any particular occupation will increase in a larger proportion, the more elastic is his marginal desire for the thing that that occupation produces.

§ 13. It remains, in accordance with the promise of § 7, to take account, in our many-commodity problem, of the fact that diminishing returns may rule instead of constant returns. The results reached in the five last sections in the main hold good; but the following additions are required. First, when Robinson's desire for one of several commodities undergoes an equiproportional enhancement, the addition to the aggregate amount of work that he does will be larger, the less sharply diminishing returns operate in respect of that commodity, and the more sharply they operate in respect of each of the others. It will also be larger the larger is the proportion of Robinson's work that was initially devoted to the commodity directly affected by the change. Secondly, when an equiproportional improvement is made in the productivity of work engaged on one commodity, the addition to work will be absolutely larger—which implies that, if there is a decrease, the decrease will be smaller—in the same conditions as above. If the elasticity of marginal desire for the favoured commodity is numerically equal to unity, there is, of course, no alteration in the quantity of work that Robinson does, no matter what degree of diminishing returns prevails or how large a proportion of his work was initially devoted to it. Thirdly, when

a uniform upward movement takes place in the whole body of Robinson's productivity schedules, it is still true that, as between commodities with similar elasticities of marginal productivity, work will be relatively expanded in respect of those of elastic marginal desire; but it is also true that, as between commodities with similar elasticities of marginal desire, it will be relatively expanded in respect of those in the production of which diminishing returns act weakly or not at all. If commodity A has a more elastic desire function, but a less elastic productivity function than commodity B, there is a conflict of tendencies. To determine the final result account must be taken, over the whole body of commodities, of both kinds of elasticity. Finally, if Robinson's aversion from work lessens, the amount of work that he does, not only on commodities of highly elastic marginal desire, but also on those of highly elastic marginal productivity (*i.e.* weakly acting diminishing returns), will expand in a specially large proportion.

CHAPTER X

THE SIZE OF ROBINSON'S CAPITAL STOCK AND THE RATE OF INTEREST IN A STATIONARY STATE

§ 1. In Chapters VIII. and IX. very stringent simplifying assumptions were made. It was postulated that existing fixed capital lasts for ever, which implies that no renewals or repairs are needed, and that there is no working or liquid capital. We have now to investigate certain important issues that arise when these assumptions are removed. The starting-point of the analysis is, it will be understood, that our state is a stationary one: which implies that Robinson is undertaking no net saving (*i.e.* net investment) and no net dis-saving (*i.e.* net dis-investment) whatever. But we now bring into account the fact that working and liquid, as well as fixed capital exists, and that all three kinds, not being inherently eternal, need to be maintained intact.

§ 2. At the first stage let us still suppose (1) that fixed capital is eternal, and (2) that every consumable good or service, once made, is instantly perishable, so that no liquid capital can exist: but let us allow that an interval elapses between the performance of work and the emergence of the good or service for which it is responsible. We thus introduce working capital. To begin with the simplest possible case, let us imagine a commodity of such a sort that the work done in respect of it one day emerges as product k days afterwards, no work whatever being done on it after the sowing of the seed. Robinson's mental make-up is such that he discounts future satisfactions in some definite measure. It follows that his desire for given quantities of commodities of delayed production when he is working to

produce them is not the same as his desire at the date of their emergence to receive them. Since we have to do with a stationary state, we must assume that Robinson will be receiving the same quantity of all commodities and will be in all other respects situated similarly at all dates. It follows that his desire *now* for the marginal unit of a given quantity of anything to be received k days hence is equal to his desire *then* for this marginal unit discounted at his rate of discounting future satisfactions in respect of k days. It is the discounted value of the future desire, and not the future desire itself, that he balances against his aversion from the increment of work that yields the marginal unit of commodity.

§ 3. The same general principle holds when the relation between invested work and resultant product is more complex. In the preceding section we supposed that a unit of product emerging now is the result of so much work engaged on it k days ago. More usually commodities, as they finally emerge, embody a series of pieces of work done on them since the beginning of their production period, it may be of equal amounts done every day, it may be of varying amounts. In these cases, for the present purpose, the situation is the same as it would be if all the work were done on the day which constitutes, so to speak, the centre of gravity of the days over which it is spread. This day, if the work is spread evenly, is evidently the one lying midway between the initial day on which any work is done on a given unit of a commodity and the day on which that unit is finished. If most of the work is concentrated near the start of the production period, the centre of gravity will lie on that side of the middle day; in the converse case on the other side. But in any event there will be some definite centre-of-gravity day, on which we may regard all the work as, in effect, concentrated.[1] There is then balanced against Robinson's desire for the marginal unit of commodity, discounted for the interval between its emergence and the centre-of-gravity day, his aversion, discounted to

[1] As Marshall has pointed out, it is not necessary that people should discount future satisfactions in simple proportion to their distance in time. If Robinson does not do this, the relevant arithmetic is made complicated, but no difficulty of principle is introduced.

that day, from all the work embodied in the marginal unit of the commodity.[1]

§ 4. On the basis of this discussion it is easy to see that the quantity of each several sort of commodity, that Robinson enjoys as income, is determined by the elements reviewed in the preceding chapter together with the rate at which he discounts future satisfactions. But this does not exhaust the subject. Given the character and the length of the period of production of any one commodity, when the (constant) annual output is given, the quantity of work that stands continually stored up in partially made units of that commodity in process of manufacture—working capital—is also given. The calculation is easily made. Thus, suppose the character of the production period to be such that the work Robinson does to-day yields its service k days hence without any further intervention on his part—that he sows the seed and does nothing further. Then, if his daily input of work is Q and if the period of production is k days, the standing stock of working capital, in respect of a daily output of commodity that embodies kQ units of work, is always k^2Q. Again, suppose that, towards each unit of commodity that will ultimately emerge, Robinson devotes daily the same constant amount of work. The standing stock of working capital, in respect of a daily output of commodity embodying the same quantity of work, is then always $\frac{1}{2}k^2$Q. These calculations are readily expressed in a generalised form.[2] But this is a minor matter. The essential point is that, given the length and character of the period of production together with Robinson's tastes and capacities and his rate of discounting future satisfactions, in order that the state may be stationary,

[1] If $f(x)$ be the work done at time x from the beginning of the production period, whose length is k, the centre of gravity of the period is at an interval r from the beginning of it, r being such that

$$\int_r^0 f(x)\,.\,dx = \int_k^r f(x)\,.\,dx.$$

[2] With the notation of footnote 1, the generalised expression for the standing stock of working capital is

$$=(k-r)\int_k^0 f(x)\,.\,dx.$$

the quantity of working capital in respect of any specified commodity can only be one single definite quantity. If it is any other quantity, the state is not stationary, but forces are at work to augment or to decrease it. In given conditions there is not a choice between alternative stationary states with different amounts of working capital in respect of the several commodities that are being produced. Only one stationary state with definite amounts of each species of working capital is possible.

§ 5. Let us next reckon with the fact that fixed capital, Robinson's hut, canoe, tools and so on, are not eternal, but, unless repaired and renewed, will decay; and also with the further fact that some consumable commodities, on their emergence to the light, do not perish instantly, but endure for a certain time, constituting, while they stand in manufacturers' or dealers' hands, liquid capital. Thus we now bring into our general picture, along with working capital, fixed capital and liquid capital also. No new principle is involved. Since we are postulating a thorough-going stationary state, these stocks of fixed and liquid capital must be supposed, both in large and in detail, to be unchanging in amount. That is to say, everything contained in them, as it wears out, is precisely replaced, but no additions are made. It is then easy to see that what has been said about working capital holds good equally of fixed and liquid capital. The quantities of each kind, when the physical environment and Robinson's nature are given, are determined in a precise and definite manner.

§ 6. Now, when there is a stock of capital, this stock yields a return which is capable of being expressed as a rate of interest. In the fluctuating conditions of real life, where the values of different commodities relatively to one another are continually varying, the analysis of this concept involves considerable difficulty. But in a stationary state, where relative values are, *ex hypothesi*, constant, there is no difficulty. The rate of interest is necessarily one and the same, no matter what the commodity is in terms of which it is expressed. Let us conceive it as expressed in money. If then we take the money cost of the marginal unit of any section of capital and divide it into the money value of the difference

made to the total annual product of that section by this marginal unit, the result is the annual rate of interest per unit. If we conceive of Robinson as ear-marking to capital that share of the total value of product which is obtained by multiplying quantity of capital by the value of its marginal yield, then this annual rate of interest per unit is also equal (in a stationary state) to the total money cost of any section of capital divided into its annual earnings. The rate of interest per cent is, of course, the rate of interest per unit multiplied by 100: and, equally of course, for equilibrium in a stationary state, this rate of interest is the same on all sections of capital. On this basis it is easy to show that the quantity of Robinson's capital stock needed to satisfy the conditions of a stationary state—a quantity which in the preceding section we showed to be uniquely determined—is such quantity that the rate of interest per given time interval is equal to the rate at which Robinson discounts future satisfactions. The proof is as follows. Let the rate of interest as defined above, and also the rate at which Robinson discounts future satisfactions, be, say, 5 per cent. In these conditions, if Robinson neither invests nor dis-invests, since, the state being stationary, he reckons to have the same income for consumption a year ahead that he has now, he will also discount *goods* a year hence at 5 per cent. There is, therefore, equilibrium. But, if he invests anything, he must withdraw income from present consumption, and must, therefore, reckon next year's income for consumption as larger than this year's: which entails that the marginal desiredness to him of goods then is smaller than that of goods now. It follows that he will discount *goods* a year hence at more than 5 per cent. But by investing he can only get 5 per cent. Therefore he will not invest. By a precisely analogous argument it can be shown that he will not dis-invest. It follows that the stock of capital will be held constant. Constancy, in short, is achieved, provided only that the objective rate of interest, as defined above, is equal to Robinson's rate of discounting future satisfactions; and no further condition is required.

§ 7. This conclusion, it will be noticed, runs counter to the opinion, which has sometimes been entertained, that in a

stationary state the rate of interest must necessarily be nil. That opinion has been vigorously sponsored by Professor Schumpeter.[1] It also appears to have been held by Wicksell. For he writes: "If we consider society as a whole, and regard the average economic conditions as *approximately* stationary, the progressive accumulation of capital must be regarded as economic so long as any rate of interest, however low, exists. Under such conditions we should expect the continued accumulation of capital—though at a diminishing rate—and at the same time a continued fall in the rate of interest."[2] This implies that a completely stationary state, with its stock of capital constant, is only possible provided that the rate of interest is nil. In the light of the foregoing argument it seems plain that this view is incorrect. It is necessary to a stationary state that the capital stock shall stand at such a level that the rate of interest is equal to Robinson's rate of discounting future satisfactions. If that rate is nil, then the rate of interest must also be nil. But that rate need not be nil. So far as *a priori* considerations go, it may be anything whatever, 50 per cent per annum or even *minus* 50 per cent per annum. What it is in actuality is brute fact depending on Robinson's mental make-up. For a representative Englishman of the present day it is probably positive and fairly small.

§ 8. The preceding section was concerned with the conditions of equilibrium in a given stationary state. We have now, following the lead of Chapter IX., to compare, this time in respect of capital stock, two stationary states that differ in the relevant fundamental particulars. This is easily done. First, if in the second stationary state Robinson's rate of discounting future satisfactions is less than in the first, the annual marginal product of the stock of capital then ruling, expressed as a rate of interest, in the manner indicated in § 6, must be equal to Robinson's new rate of discounting future satisfactions. In accordance with the rule of diminishing returns to individual factors of production, of which more will be said in later chapters, this implies that the stock of capital is larger in the new stationary state than in the

[1] *The Theory of Economic Development*, chapter 5.

[2] *Lectures on Political Economy*, p. 209.

old. This implies that Robinson's annual income is larger. Secondly, if in the second stationary state there is an improvement, which makes *any* quantity of capital associated with a given quantity of work more productive at the margin than it was before, but does not affect the productivity at the margin of any given quantity of work associated with a given quantity of capital, this also necessitates an enlargement in the stock of capital—an enlargement sufficient to bring the rate of interest down to the level of Robinson's rate of discounting future satisfactions. As before, Robinson's income will be larger in the second stationary state than in the first. Thirdly, if in the second stationary state there is an improvement, which makes *any* quantity of work associated with a given quantity of capital more productive at the margin than it was before, but does not affect the productivity at the margin of any given quantity of capital associated with a given quantity of work, no effect is produced on Robinson's stock of capital; but his income is enlarged. Finally, if in the second stationary state Robinson's aversion attitude towards work is less strong than in the first, the marginal product of any given stock of capital is bound to be enhanced in consequence of the extra work that he does. It follows that the stock of capital must be larger in the second stationary state than in the first; and Robinson's income must also be larger.

§ 9. So far we have held to the assumption, as set out in § 1 of Chapter IX., that Robinson's productive capacity and aversion from work are both independent of the amount of his consumption, that is, in a stationary state, of his income. When this assumption is abandoned, the consequences of the four types of change distinguished in the preceding section, all of which entail enhancements in Robinson's income, may be modified. For, if, with an enhancement of income available for consumption, Robinson's capacity is increased more than his hours of work are contracted, the marginal productivity of any given stock of capital is thereby made larger than it would otherwise be. In all the four cases distinguished above the stock of capital will then be larger in the second stationary state than it would be if the assumption so far made

were valid; and Robinson's income will, therefore, also be larger. In converse conditions the opposite results follow.

§ 10. The conclusions set out in the two last sections are all valid provided that, as we have so far tacitly supposed, Robinson's rate of discounting future satisfactions is the same in the second stationary state as in the first. If this is not so, but enlargement of income leads indirectly, through a change in his mental make-up, to a lowering of this rate,[1] the argument throughout is *a fortiori*.

[1] Cf. *post*, Chapter XXXII. § 5.

CHAPTER XI

THE ASSUMPTION OF PERFECT DIVISIBILITY

§ 1. In the study of a Crusoe economy conducted in the three preceding chapters it has been assumed [1] that both Robinson's work and the various commodities that he obtains by means of it can be divided, without cost or change of character, into indefinitely small parts. Unless this assumption is made analysis by way of margins always leaves a rough edge. Since throughout what follows this type of analysis will play a large part, it is desirable to obtain a clear understanding of the matter.

§ 2. When Robinson exchanges with Nature work against apples, if there is no limit to the divisibility—without cost or change of character—of both these things, he will go on working till his aversion from the marginal unit of work exactly balances his desire for the apples that result from this marginal unit. But, if divisibility is not perfect, Robinson will barter n units of work for apples, provided that (1) his aversion from the nth unit of work is less than his desire for the yield of apples due to the nth unit, and (2) his aversion from the $(n+1)$th unit of work is greater than his desire for the yield of apples due to that unit. There is thus, not a single precise condition of settlement, but a finite range of possible conditions. The same thing is true if, for the moment abandoning Robinson, we imagine two people A and B, of whom A is exchanging a units of his stuff against b units of B's stuff by way, not of successive bargains, but of a single rate of

[1] Cf. *ante*, p. 35.

exchange. We know that the ratio a/b cannot exceed the ratio,

$$\frac{\text{A's desire for least desired unit of B's stuff}}{\text{A's aversion from parting with least desired } a/b \text{ units of A's stuff}},$$

and cannot fall short of the ratio,

$$\frac{\text{B's aversion from parting with least desired } b/a \text{ units of B's stuff}}{\text{B's desire for least desired unit of A's stuff}}.$$

If our units are perfectly divisible, these two ratios must, for equilibrium, be equal: but, if they are not perfectly divisible, a finite gap may exist between the ratios. Obviously the size of the gap is greater, the larger are the units in use; and as these units contract, its size approaches continually to nil.

§ 3. It follows from this that the scale of the error that is introduced when units that are in fact not perfectly divisible are treated, for the purposes of analysis, as though they were, is serious or trivial according as the actual units of real life are of substantial size or are very small, in the sense that the number of them involved in any normal transaction is large. In the latter case the range of indeterminateness is very narrow and the error is negligible.

§ 4. It is for this reason of interest to note that many articles are, for the purposes of economic analysis, much more divisible than they might seem to be at first sight. Thus houses, suits of clothes or books cannot be split up *physically* without losing their character and their serviceability as economic entities. They can, however, be split up economically into very small parts without any change of character by means of the circulating hire system. For this enables them to be sold, not in respect of their whole duration, but in respect of successive fractions of it.[1] In general, apart from a few exceptional cases, such imperfect divisibility as there is, whether among commodities or among factors of production, may safely be ignored, at all events for the economics of a stationary state.

[1] Cf. Jevons, *Principles of Economics*, p. 7.

CHAPTER XII

MULTIPLICITY AND INTERACTION

§ 1. In the four preceding chapters we have studied the economic life of a single Robinson Crusoe isolated on his island. When for a single Robinson we substitute a large number, it is possible in point of logic that each of them may, nevertheless, act as though he were completely isolated, so that we have merely to repeat the analysis that has already been worked out. This does not, of course, imply that all the Robinsons will act exactly alike. Their tastes, the quantity and quality of their capacities and the natural resources surrounding their dwelling-places are likely to be different, and, if so, the amount and kind of work that they severally perform and the amount and kind of their several real incomes will also, in general, be different. But it does imply that the situation of each Robinson can be fully described without reference to the existence of any of the others. This situation, though logically possible, does not in fact ever arise. There are always a variety of motives at work to promote *interaction* of one sort or another among some or all of the Robinsons.

§ 2. At first sight it might seem that the only sorts of interaction that can have relevance to our subject are those that involve co-operation in production or transfer of services or goods from one Robinson to another. This is not so. Even though no such co-operation or transfer takes place, the economic conduct of the Robinsons may, nevertheless, be notably affected. The mere fact of there being a number of them instead of one opens the way for forms of desire and forms of activity that would otherwise be impossible. People

may now find a value in instruments of hospitality, of ostentation and of collective sport, that for an isolated man would have been meaningless. Robinson alone might, indeed, conceivably have wished for a golf club, but it is incredible that he should have wished for a cricket bat. Mrs. Robinson —if we may so far modify our assumption of singleness as to assume her presence in the background—could, by no possibility, have desired her husband to build for her a ballroom. Nor is it likely that her enthusiasm for diamonds would have been so keen had it not been for a wish to outshine Mrs. Jones. Thus a whole collection of new desires is created by the mere fact of a number of people being present. This clearly must cause the work they do to be turned in new directions. A substantial part of it will now go to the making of sport articles, articles that assist social entertainment and articles whose merit consists in the fact that other people know that one possesses them. All this suggests further that the colony of Robinsons, besides working along a larger number of different lines than they would do in isolation, will also do more work in the aggregate. That, however, is not certain. For leisure has now greater attractions than before; a time free from work is needed to make use of the various sorts of new articles that I have named. It is conceivable, for example, that the 5000 young men concentrated at Cambridge would do more work and not less if they were segregated upon 5000 separate islands!

§ 3. Moreover, it may happen, when a number of people are neighbours, even though there is no direct intercourse between them, that work done by one of them modifies the conditions of work, and so the yield of work, for others. If A breeds rabbits, B's work in growing vegetables may be rendered less effective. *Per contra*, if A plants a forest, he may indirectly affect the climatic conditions in a sense favourable to B's crops. Even though A and B are both doing precisely the same kind of work, they may mutually affect one another. On the one hand, they may get in one another's way as they walk about the fields; on the other hand, the walking of one may beat down a track, which makes walking easier for the other. The former of these two kinds of interaction is, of

course, much the more important. For, with land limited in quantity, the fact that A is occupying some of it *inevitably* means that there is less left to serve the purposes of B.

§ 4. Though interactions that do not directly or necessarily entail either co-operation in production or transfer of services or goods thus play some part in economic life, those that do entail these things play a much larger part. These fall into three groups, transfer by ordered authority, transfer by what we may call, for want of a better name, anarchic exploitation, and co-operation or transfer by voluntary arrangement. The main purpose of this chapter is to indicate the general character of these three groups.

§ 5. When any substantial number of people are assembled fairly close together, they are certain to experience a need for some form of governing authority, if only to maintain peace and order among them. This governing authority —it is immaterial for the present purpose whether it is autocratic or representative—is bound, in the exercise of its functions, to enforce some minimum amount of transfer to itself from the people under its authority. There are three ways in which it may do this. First, it may directly require particular personal services from individual citizens, or may commandeer particular pieces of their property. Thus it may conscript them for the army, or for a labour battalion or for a jury. The only form of direct action along these lines that is important in times of peace in States outside Russia is military conscription. In Russia, it appears, forced labour also is sometimes made use of for other than military purposes. Secondly, a government may levy taxes upon the general body of its citizens and use the proceeds to hire labour or purchase goods. When it acts in this way it enforces transfers to itself by indirect, instead of direct, process. Where military conscription does not exist, a certain number of men will have to be engaged for the army on that plan. In any event a certain number will have to be so engaged in a modern State for the police, judiciary and central and local administration. In some countries and at some times this exhausts the field of government service, but in other countries and at other times the field is much wider. In this

country public education and public health services absorb large sums every year. Thirdly, a government may levy taxes upon one set of people and use the proceeds in making payments to another set, *e.g.* in War Loan interest, pensions, contributions to unemployment benefit and so on. Here again transfers are effected by ordered authority through an indirect process; but in this case the choice and control of the transferred objects is left in private hands.

§ 6. The most obvious form of interaction by exploitation is the exercise of force by one Robinson against another. He may hold up his neighbour and compel him to do work for him, either occasionally or throughout his life, as a slave. He may make raids upon the crops he has planted or the grain he has stored; he may play the part of a highwayman or an armed burglar; or, more systematically, he may compel some proportion of the fruit of other men's labour to be surrendered to him at regular intervals. A second form of exploitation is exploitation by fraud. It appears under a régime of contract when A, by deceiving B, induces him to accept a bargain much less favourable to himself than he would have accepted had he known the facts. Fraudulent company promoters and the sellers of many kinds of patent medicines exploit their fellows along these lines. A third form is more subtle. It consists in A's threatening to create, or actually creating, a situation unpleasant to B and then allowing B to buy him off. This form is best illustrated by the art of the blackmailer. He gets his living by not doing something that B would dislike and being paid for his "abstinence". An alternative form is for A deliberately to put an obstacle in B's way and to induce B to pay him for service done in removing it. At a certain spot in the Lake District, by which a road passes, there lie the discarded remnants of a disused gate. In holiday seasons, when motor traffic abounds, the village children place this gate across the road and stand by to open it, thus "facilitating"—in the confident expectation of pennies—the passage of unsuspecting motorists.

§ 7. Interaction of this type, in general, implies the doing of work by one set of persons to bring about the gratuitous, or, at all events, enforced, *transfer* to them of other people's

wealth. Indirectly it also, in general, causes these other people to do less work, and so to produce less stuff, than they would otherwise do. Farmers in districts infested by bandits are not likely to cultivate their land as earnestly as they would do if the fruit of their labour were sure to come to them. Sometimes, however, it happens that exploitation makes the exploited work harder and produce more than they would do otherwise. This reaction is more likely to follow from a regular and fairly foreseeable exploitation, *e.g.*, through a tribute system, than from one that acts sporadically. In any event, looked at in the broad, anarchic exploitation is plainly adverse to the aggregate interest of the community in which it is practised. Consequently, all communities that have succeeded in establishing organised government make use of public authority to mitigate or suppress it. A large part of the legal system of modern States is concerned with penalising attempts at private self-enrichment through the exercise of force, fraud and blackmailing activities. The law on these matters is continually being modified to prevent evasion; and the police system is organised to prevent open breach. As a consequence, *inside organised communities* the part played by exploitation is continually decreasing in importance, though there are still certain types of activity undertaken by large business concerns against their trade rivals—boycotts, destructive rate-cutting, freezing-out—that have not been brought fully under social control. *Between* organised communities, however, interaction by exploitation in one form or another plays a much larger part than it does inside them. Until the establishment of the League of Nations and the signing of the Kellogg Pact force and the threat of force—war and diplomatic blackmail—were not illegal as between sovereign States and were not subject to any penalty. In recent times some advance has been made in this matter; but very much has still to be achieved.

§ 8. There remains interaction through voluntary private dealings. Under this head are included all ordinary private purchases, whether against prompt payment or against promises of payment later on; all private loans; all ordinary hirings of labour and of land. This vast mass of "business",

most of it conducted between citizens of the same State, but a substantial part international in character, is, of course, in complex modern communities built upon a foundation of legal rules about property and contract, interpreted in each State by the appropriate set of Courts. In the great majority of cases this machinery is never invoked, but its presence in the background, alike in fact and in men's minds, is essential to the smooth conduct of affairs. In the actual world interaction of this type is much more important than interaction by exploitation. This is not the consequence of any external Fate. The predominance of voluntary private dealings is, on the contrary, largely the result of an elaborate system of laws and institutions deliberately devised to bring about that very result. In this book nothing more will be said about interaction by exploitation. In accordance with established tradition it is postulated that, apart from the spending and raising of revenue by public authorities—to which only a passing reference will be made [1]—all economic interaction is by way of voluntary private dealings.

[1] Cf. *post*, Chapter XLVII.

CHAPTER XIII

THE CAUSES AND CHARACTERISTICS OF VOLUNTARY PRIVATE DEALINGS

§ 1. THE causes of voluntary private dealings consist, of course, in the advantages which, under the aegis of State rules about property and contract, those who engage in them expect to gain. The present chapter will, therefore, consist of a general account of what these advantages are. The essence of what has to be said was set out long ago in Adam Smith's masterly discussion of the division of labour, and is too familiar to warrant lengthy exposition.

§ 2. Voluntary private dealings include co-operation in production and exchange. When we think of production as a whole, these two things appear as opposite sides of the same shield. People co-operate by producing services and exchanging them, directly or indirectly, for other services. It is, however, convenient to draw a distinction, within the broad group, voluntary private dealings, between the exchanging against one another of finished commodities and co-operation by different people in making each of these commodities. This distinction is not, indeed, one that should be pressed unduly. For "finished commodity" is an ambiguous concept. A cheese, for example, in a consumer's house, though commonly conceived as the same finished commodity as a similar cheese in the factory, in fact differs from it by the transport and trading services that it embodies. Still, provided that its logical imperfections are recognised, the distinction helps exposition.

§ 3. In a community where, for physical or other reasons, co-operation between different people in the production of

individual commodities was impossible, there would still be a great gain to total output if, instead of everybody making for himself all the commodities that he wanted, each specialised in making one or a small number of commodities, and entered into exchange relations with his neighbours. For continued practice over a narrow range enormously increases skill; and, the fewer the transitions that have to be made from one kind of work to another, the smaller is the wastage of energy and time. Of course there are limits, set by technical conditions and by the numbers of the community, to the extent to which specialisation and exchange in this sense can profitably be carried. If there is some article, of which everybody wants such a minute quantity that the total quantity wanted would not take even one man all his time to make, some people, if not all, will have to make this article as well as their main commodity. But in a broad general way we may say that gain will result from everybody concentrating on a single sort of commodity. By doing this and exchanging among themselves their respective surplus products, they will all, for a given amount of work, obtain more than they could have done otherwise of all those commodities, some quantity of which they would have obtained anyhow; while, over and above this, labour power will be set free to procure for them a number of other commodities, which, apart from specialisation and exchange, they would not have had at all.

§ 4. The advantages yielded by co-operation in the production of particular finished commodities are of a like general character. Where it is practicable, specialisation can be pressed beyond completed commodities to small parts and small processes; skill being enhanced and waste of energy and time in transitions being saved at every stage. The number of operations, into which the making of a finished article can be broken up, varies, of course, with the technical situation. "A pair of shoes is now made by groups of men working at some 240 different operations. The manufacture of watches, typewriters, motor cars is broken up into many tiny parts. The old model T Ford is the product of 7882 different jobs. More simple products are incapable of such

fine subdivision. A table knife or a loaf of bread could not with advantage pass through a thousand hands; however large the output to be produced, a dozen could do the work as well. But a motor car could not be cheaply manufactured by a dozen men however skilful."[1] Moreover, in some cases different operations cannot advantageously be allocated to different workers on account of the great difficulty that would be involved in co-ordinating their work. Thus, though it would be physically possible, it would not be advantageous to have a number of different men engaged in painting different square inches of a portrait. Difficulties of co-ordination, however, though they limit, are far from doing away with, the advantage for production of co-operation by many different kinds of specialised workers. If there are, so to speak, enough men to go round, the logic of the situation is that, when they once start co-operating with one another, the whole set of them, originally Jacks of all trades, presently find themselves, because by so doing they can get more stuff for less work, becoming specialist exponents of single jobs or small ranges of jobs. This would be so even though initially all the men were exactly alike. *A fortiori* it is so in so far as their inborn aptitudes are various.[2]

§ 5. To this cursory account—every reader should go for himself to Adam Smith's masterpiece—a concluding word may be added. In an economy where there is exchange of finished commodities, but no co-operation in making them, we may properly allow ourselves to reckon as finished commodities the services of transportation and of wholesale and retail dealing. These, to the great gain of everybody, will have their specialist workers. But in an economy where co-operation in the making of individual finished goods is admitted there is something more. A new type of specialist is required, one who stands at the hub of the economic wheel.

[1] E. A. G. Robinson, *The Structure of Competitive Industry*, p. 20.

[2] It may be noted that, if they were all initially exactly alike, a state of non-co-operation would be in equilibrium. "Comparative costs" would be everywhere equal. But the equilibrium would be unstable: for, if, accidentally, any degree of specialisation were introduced, there would be no tendency to return to the original position, but a cumulative tendency to depart further from it.

If a number of people were making, each of them individually, complicated objects ready for consumption, it would be possible, though highly inconvenient, for them to interact with one another by direct barter without any mediation. But, when different people are providing services which separately do not yield completed objects, that is not feasible. Suppose, for example, that a number of different kinds of hand and brain workers, assisted perhaps by machines owned by yet other people, work together in an integrated concern that turns raw cotton into finished cloth. Robinson A, Robinson B, Robinson C and so on all want finished cloth. None of them can be satisfied by the bartering of an hour's spinning work, either of themselves or of machinery belonging to them, against an hour's weaving work. What is required is that a certain proportion of the cloth that results from their combined operations shall somehow be allocated to each. This implies the presence of some person to make bargains with each of the several Robinsons. These cannot deal together directly. Their interaction must, in general, be accomplished indirectly through a mediator, who pays out wages, interest and rent for the services of his various human and material assistants. This mediator may be either an independent person who hires the labour, capital and land, or himself the hired agent of those who provide one or other —usually the second—of these factors. It is important to bear in mind, however, that, in a stationary state this functionary, whether he be hirer or hired, differs fundamentally from the *entrepreneur* of the actual world, in that he does not need to introduce innovations, to undertake risks or to forecast an uncertain future.

CHAPTER XIV

THE NEED FOR MONEY IN VOLUNTARY PRIVATE DEALINGS

§ 1. BEFORE the more complex problems associated with voluntary private dealings are attacked it will be convenient to review in turn one *set of arrangements*, a money system, without which co-operation and exchange cannot function effectively, and one *concept*, that of a market, without which their functioning cannot be understood. This chapter is concerned with money, Chapter XV. with markets.

§ 2. In a community consisting of one person only there is obviously no need for money. Nor is there need for it in a community consisting of many persons, so long as only a single sort of commodity is being produced. For in such a community some of the functions normally performed by money could be performed by this commodity itself, while other functions would not need to be performed at all. Thus little purpose would be served by the emergence of a specialised medium of exchange. No doubt, we could, if we wished, arrange for paper notes to be paid out to workpeople and shareholders, with which these people should then buy their appropriate share of the one commodity. But it would be almost, if not quite, as easy to arrange for them to be paid directly with the commodity itself. The second principal service usually performed by money, that of constituting a unit of account, or measure of value, in terms of which different commodities can be given a single price, and so obviating cumbrous arrangements for pricing each of them in terms of each of the others, will obviously have no place in a one-commodity community. The third and fourth

services, those of providing a standard of value for deferred payments and a store of value, could no less obviously, in such a community, be adequately performed by the one commodity itself. But in an economy of co-operation and exchange, containing, not merely many persons, but also many different sorts of commodity and service, the case is quite different. It is not merely that, for the exchange of goods against goods, double coincidence would often be lacking. Payments for the services rendered by the factors that co-operate in complex forms of production would be still more difficult to carry through. For each captain of industry to exchange the commodity produced by him for other commodities in the exact proportions needed to satisfy his various payees would require in him an impossible degree of knowledge about their several tastes, besides rendering every individual act of wage payment an intolerably lengthy business. If, to avoid this, each captain paid his assistants in his own product and left it to them to exchange this by barter for the things they wished to consume, there would again be hopeless difficulty. The inconvenience would be serious enough if the only products made were articles divisible into small units, such as bread, and beer, and boots. But many commodities are not thus divisible. A concern whose business it was to build Atlantic liners or other objects of large individual value would be debarred absolutely from arrangements of this kind. It is impossible to pay out as wages bits of an Atlantic liner. But there is no need to repeat here what has so often been said on these matters. It is obvious that a community based on co-operation and exchange would suffer serious inconvenience if there were no instrument to perform the several services that have been described above.

§ 3. It is not logically necessary, as Jevons long ago made clear, that all these services should be performed by the same instrument. Among the actual concrete media of exchange currently employed there need not be any whose name is identical with the name of the pricing unit used in prompt, or of that used in deferred, transactions; nor need these two pricing units be identical. Thus it might happen

that all goods were priced in guineas and fractions of guineas, though no actual guineas existed. It is not even essential that the unit of pricing—say the guinea—should have a stable value in terms of the current medium of exchange. For example, in the post-war monetary chaos in Germany goods in shops were often priced in dollars, while the number of marks that a dollar represented was continually changing. But, of course, it is convenient that abstract pricing units for prompt and for deferred transactions should both coalesce with one of the concrete units embraced in the system of exchange media. In all ordinary circumstances in modern States this requirement is satisfied. Furthermore, the media of exchange, while held in anybody's hands, necessarily also constitute for him a store of value in readily available form. Thus the monetary system is a single coherent whole.

§ 4. In the fluctuating conditions of actual life the role played by that system is extremely complex, and is the proper subject matter of prolonged inquiry. But in a stationary state it is comparatively simple. In so far as economic affairs are conducted through the mediation of money, every real transaction is accompanied by a monetary transaction—a shadow, if we will, of the real one. In actual life these real transactions include capital transactions, in which pieces of durable property change hands against money—including, of course, bank money—from time to time. Here money is on a par with everything else. It is one sort of property among others. The things against which it is successively exchanged have the same degree of permanence that it has. But in a stationary state there are no capital transactions —only income transactions. In these the relation between reality and shadow is a different one. A succession of evanescent entities appear, are exchanged against money and then disappear. Money—by which I shall here mean *relevant* money, to the exclusion of mere hoards of money that never enter into money income at all—is a permanent stock. It stands available to perform the service of mediating exchanges for innumerable generations of new income objects. There is not a stream of new money flowing into the lake of being from nothingness at one end and out again into

nothingness at the other, as the stream of real income does. On the contrary, the same money that flows out of the lake returns again and again to flow into it. Money is always entering along with the real income that enters, and always going out along with the real income that goes out. But, whereas real income, after passing through the lake, disappears for ever, money repeats its journey again and again in endless succession. There is an annual flow of money income paid out to the providers of real income for their services and a fixed stock of money. In stationary conditions the annual flow and the stock are both constant, and there is, therefore, between them a constant numerical relation. The annual money income of the community is such and such a multiple of its fixed money stock.

§ 5. The general character of the process, by which, on the basis of the fixed stock, an endless succession of equal money incomes is generated, is clear enough. People on every pay-day receive a certain money income, which they presently spend in buying goods from shops and personal service from professional men and domestic servants. The shopkeepers pay out a large part of what they receive to the wholesalers with whom they deal. The net balance of their receipts from sales over their expenditure in purchasing the things to be sold and the other costs of running their business constitutes their money income. A like net balance constitutes the money income of wholesalers, of manufacturers and of everybody else. Thus the total money income per year of the community must by no means be confused with the gross turnover of money, which is a quite different and much larger sum. This total money income, having been received, is presently spent, and, in the process, generates new money income. If we suppose that, for all units of money that enter into income, the interval between receipt as income and expenditure *plus* the interval between expenditure and reappearance as income have always the same duration, the succession of money incomes that are generated in successive periods of time must all be equal to one another; and also, for periods of the same length as the interval between successive appearances of the representative

money unit as income, must be equal to the money stock. In actual life, of course, the intervals between receipt and expenditure are different for different incomes, and the intervals between expenditure and resultant income are also different. A man whose income is paid to him once a quarter does not, on the average, pay it out again so rapidly as a man on weekly wages. Nor do different types of expenditure generate new incomes with equal speed. If I buy the services of a professional man, my expenditure becomes his income immediately; but, if I buy a Rolls-Royce car and the company several months later pays out to its shareholders the part of the selling price that is profit, the interval between my expenditure of money income and the reappearance of this part of it as shareholders' income is considerable. Thus we have not to do with a simple situation, in which all the money items that are income on any day become income a second time after one and the same number of days. To-day's income is reincarnated, some of it to-morrow, some of it a week hence, some of it a year hence. The income of any day is thus generated, not from the income of one earlier day, but from parts of the incomes of many earlier days. But these different parts, some reincarnated after a long interval, some after a short one, must always add up to the same amount, provided that the proportions of the money stock that enjoy different intervals between successive incarnations remain constant, and that, over a period long enough to cover the longest interval, daily incomes have been equal. This analysis might, of course, be carried much further; but to attempt this would be to wander from our present purpose.[1]

§ 6. In the two preceding sections a very brief and summary account has been offered of the way in which in a stationary state the money system functions. To some readers that account may perhaps suggest that money in such a state plays a purely passive role, merely providing a method for carrying through transactions that would have taken place by *some* method whether money had existed or not. It may be well explictly to repudiate this error, lest the implicit repudiation contained in § 2 may not have been fully

[1] Cf. *The Theory of Unemployment*, Part IV. chaps. ii.-iv.

grasped. The error is a very serious one—exactly similar to the error which would be made if anyone were to say that a highway is merely used for acts of transport, which, had there been no highway, would in any event have been accomplished somehow. The analogy is complete, for, as Adam Smith observed long ago, the machinery of money and credit provides, in effect, a highway for commerce. The difficulties and obstructions that would inevitably be met with under a régime of barter have been repeatedly emphasised by economists. Obviously, just in so far as the absence of a money system handicaps many types of exchange, the presence of one facilitates them. It enables a much higher degree of specialisation and division of labour to be established than could come about in a barter economy, and thus makes possible either increased leisure—the establishment of shorter hours of labour—or a higher scale of consumption, or, it may be, both of these things together.

CHAPTER XV

MARKETS

§ 1. To enable the functioning of an economy that embraces co-operation and exchange to be properly understood we need, as was stated at the beginning of the last chapter, the concept of markets. *Prima facie* among these markets some will be perfect and others imperfect in varying degrees. I propose, following Jevons, to define perfection for markets in a special way. "A market", he wrote, "is theoretically perfect when all traders have perfect knowledge of the conditions of supply and demand and the consequent ratio of exchange."[1] This use of terms, which is not, it must be confessed, always consistently maintained by Jevons himself, is by no means universal among recent writers. Some of them speak of markets as imperfect, not only where ignorance prevails, but also, even apart from this, where any element of monopoly is present. For me, however, monopoly, equally with competition, is compatible with market perfection. Any market in which there is no ignorance is a perfect market. But in a stationary state ignorance may properly be conceived as smoothed away. Hence the markets with which we have here to do are all perfect—or at least approximate to perfection so closely that the measure of imperfection they still retain may be disregarded. For the purposes of this discussion, therefore, a market *means* a market that is perfect. There is for us no other kind.

§ 2. The term market is sometimes used in such a way that any two articles are regarded as in a common market if the actual price anywhere of one of them is different from what it

[1] *The Theory of Political Economy*, Third Edition, p. 87.

would have been had the other not existed. This condition is obviously satisfied when two articles, different in kind, but in some measure capable of satisfying the same needs, are on sale in the same place. It is likewise satisfied when the two articles, instead of being rivals, are complementary to one another. Yet again it is satisfied when there is a linkage, not on the side of demand, but on that of supply; whether the two things are jointly supplied or are competing users of some raw material. Nor is even this all. In a strict sense the condition is satisfied for all different kinds of articles that are bought and sold in the same neighbourhood, even though they are not related in any of these special ways. For, if one ceases to exist, and, therefore, no money is expended on it, there is more money available to spend on the others and their prices are affected. This means that everything in the same neighbourhood is in a common market, and the word loses all significance.

§ 3. To obviate this result, I shall premise that in one market there can only be one single type of (homogeneous) commodity or service.[1] Commodities that are good substitutes for one another are not, as is sometimes suggested, in "an imperfectly competitive market".[2] My use does not allow this. In respect of any number of different commodities or services, however good substitutes they may be, there are at least that number of different markets. For different types of motor car there *cannot* be one market, but only a number of closely interconnected markets. This, of course, is a matter of words, not of principle. I state my proviso without attempting to prove that the plan followed here is "better" than any other plan.

§ 4. Having thus decided that no units of different articles can be in one market, we have to inquire next in what

[1] From the standpoint of a short period it may often happen that different units of a physically identical commodity are not homogeneous from the standpoint of the buyer; *e.g.* he prefers to buy the units offered in some particular shop. In a less degree this may also happen from the long-period standpoint of a stationary state. To avoid complicating the argument, I shall, however, ignore it. Anyone who wishes to do so can bring it into account, without departing from the general plan followed in the text, by regarding it as a kind of virtual cost of transport (cf. *post*, Chapter XLIV. § 8).

[2] Chamberlin, *The Theory of Monopolistic Competition*, p. 68.

conditions different units of a single homogeneous commodity respectively are and are not in one market. The answer is that different units of it are in one market provided that, *en route* between the sellers' works and the buyers' home, they pass through, and are available for purchase and sale to any applicant at, a common nodal place. A nodal place is conceived as a region between the several parts of which costs of transport are nil. Thus, if costs of transport were in fact nil everywhere, the whole world would be embraced in the same nodal place. But in actual life, where costs of transport are not nil, nodal places are contracted to very small areas; in effect, they are nodal points. This definition, it will be noted, allows the same unit of a commodity to be at the same time in two or more different markets. For there is nothing to prevent it, in its journey from works to home, passing through and being available at the nodal points of any number of separate markets.

§ 5. These nodal points must be situated somewhere in space. If there is only one seller and only one buyer, the position of the relevant nodal point is ambiguous. It may be regarded indifferently as lying anywhere on the line of cheapest transport from the seller's works to the buyer's home. If there is only one seller, but more than one buyer—apart from the special case of the line of cheapest transport to the most distant home passing through all the homes—it must lie at the seller's works; in the converse case at the buyer's home. In general the nodal point of any market will occupy some sort of central position among the mass of works and homes that it serves. If there is a number of works scattered at different distances round it and a number of homes also scattered at different distances, it may lie very close to one of the works, or to one of the homes, or to both a works and a home, or it may lie a long way from any works or any home. It does not matter where it is. The essential fact is that all the units of the commodity that are in the market, of which it is the hub, must pass through it.

§ 6. A hasty reader may be tempted to conclude that, if this is so, most markets will be of very narrow range, *i.e.* that for most commodities there will be a large number of small

separate markets. For, he will urge, in practice it will usually not pay to send stuff to be dealt in except at a place that lies more or less on a straight line between the originating works and the purchasing home; and not many of these straight lines are likely to pass through any one point or small area. This reasoning, however, ignores the fact that transport over land can only be accomplished with reasonable cheapness if it follows prepared ways, and that the construction of prepared ways is expensive. The implication of this will be at once realised if we mentally contrast a telephone system under which each subscriber is directly connected with all the others with one in which the connections are made through a central exchange. Suppose that there are four subscribers situated at the four corners of a square. For indirect connections through the centre of the square a line is required equal in length to twice the diagonal of the square; for direct connections a line equal to twice the diagonal plus four times a side of the square. With a large number of subscribers it is obvious that the extra total distance to be traversed with a system of direct connections, and so the extra cost, would be enormous. A comparatively small number of individually large distributing centres *must* be established. For a like reason the individual units of most commodities, in the course of their journey from works to homes, must be concentrated in large numbers at one or another of a *few* points. This is, indeed, the reason why wholesale and retail establishments exist at all. Markets are thus at once fewer in number and of wider individual range than might at first sight be supposed.

§ 7. But this does not exhaust what has to be said. In § 4 there was laid down as a condition for different units of a commodity being in the same market, not merely that in their passage from works to home they should pass through the same nodal place, but also that they should be available there for purchase and sale to any applicant. It is not enough that the several units should all pass through a nodal place in the physical sense, *e.g.* in a rocket aeroplane. The second condition, availability, is also essential. Different units of personal service, *e.g.* operations for appendicitis, transport,

and commodities such as water, gas and electricity, which have to be delivered to their purchasers through a private apparatus, are not in the same market, however close to one another their respective purchasers may live.

§ 8. In a market containing only one buyer and only one seller, who bargain by way of total amounts exchanged, questions about the relationship of the prices of different units of the commodity traded are meaningless. When A, for example, exchanges with B 100 herrings for 100 tomatoes, the units of tomatoes and herrings have a definite average price, but no distinguishable specific price. In these conditions there is nothing to be said. Provided, however, that A and B bargain by way of a rate of exchange, as, for example, the representatives of a Trade Union and an Employers' Federation do, there is a specific price, and this price can, of course, be only one. When there is more than one buyer or more than one seller, there is not in any case any difficulty. At the nodal point of any (the same) market in which they deal all units of commodity that are bought and sold must have the same price. For, if any seller offers to sell at less than the common price, the purchases of all the buyers will be concentrated upon him, the other sellers being deserted, until price uniformity is restored. In like manner, if any buyer offers to buy for more than the common price, all the sellers will concentrate upon him and desert the others until, as before, price uniformity is restored. In short, any difference between the prices of different units of a single homogeneous commodity—*i.e.* a commodity such that, in Jevons' words, "any portion may be indifferently used for any other portion"[1]—at the nodal point of a market automatically evokes processes that tend to, and so, in the conditions postulated for a stationary state, do in fact remove it.[2]

[1] *Theory of Political Economy*, p. 91.

[2] The pseudo-converse proposition, "Equality of price between two points implies that the units of commodity sold there are in the same market", is not, of course, valid. Between two places separated by transport costs, say, London and the capital of Mars, the gold prices of tomatoes may by accident be identical, but the two sets of tomatoes are not in the same market.

§ 9. As between the nodal point and other points in the same market price relations are as follows. If there is no cost of transport (including, of course, tariff charges and so on) between the nodal point and the homes of the buyers, price uniformity at the nodal point implies price uniformity at all the homes. Similarly, if there is no cost of transport between the nodal point and the works of the sellers, it implies price uniformity at all the works. But, if there is cost of transport along these routes, it does not imply that. It implies that at each home price is equal to price at the nodal point *plus* cost of transport to that home from the nodal point, and that at each works it is equal to price at the nodal point *minus* cost of transport from the works to the nodal point. I am, of course, assuming that transport from any works to any home via the nodal point is always charged an amount equal to the sum of the charge from the works to the point and the charge from the point to the home.

§ 10. As between any two markets, if any unit of commodity is in both markets, in such wise that, *en route* from its seller to its buyer, it passes first through the nodal point of market A and *afterwards* through that of market B, being available for purchase and sale at both, the price per unit at the nodal point of B must *exceed* that at the nodal point of A by the cost of transport of a unit from the nodal point of A to that of B. Since the prices at all other points in A are related to that at its nodal point in the manner described in § 9, and similarly with B, the relations among the prices ruling at all the several points in the two markets are thus determined. If there is no unit of commodity in both markets in the above sense, and if there are not in the two markets any points between which passage can be made more cheaply than between their respective nodal points, the price at the nodal point of A cannot exceed that at the nodal point of B by more than the cost of transport from the second nodal point to the first, or fall short of it by more than the cost of transport in the opposite direction: but within this range the price relation between the nodal points is, on the data, indeterminate. This, of course, entails a like range of indeterminateness in the price relation between any specified works

or home in the one market and any specified works or home in the other. If there are points in the two markets between which passage can be made more cheaply than between their respective nodal points, the prices at the two nodal points cannot differ by more than the cost of the cheapest passage between any point in the one market and any point in the other *plus* the sum of the costs of transport between each of these two points and the nodal point of its own market. The range of indeterminateness *may* thus be narrowed.

§ 11. There remains the case in which one seller disposes of some of his product through the nodal point of one market and some through that of another; and the analogous case of a buyer who purchases through the nodal points of two (or more) markets. Obviously there is nothing to prevent these things from happening. If the seller or buyer in question acts on the rule of competition, *i.e.* accepts the price of each market without reckoning on the possibility that their prices may be affected by his action, such cases can only arise provided that the difference between the prices ruling at the two (or more) nodal points is exactly equal to the difference in the costs of transport between these nodal points and the seller's works or buyer's home ; *e.g.* in the limiting case, when these costs are equal, provided that the prices at the two nodal points are equal. For, if a seller or a buyer finds one of the nodal points more advantageous to deal in than the others, he will confine his dealings wholly to that point; or, alternatively, will press his dealings at that point and withdraw from the others till price differentiation disappears. If, however, the seller or buyer in question acts on the rule of monopoly, he may deal through the nodal points of two or more markets without it being necessary for the prices at these points to be related in the above manner, *provided that the difference between the prices ruling at the two nodal points is less than the cost of sending units dealt in at one of them to the other, to be dealt in again there.* When the costs of transport from a monopolist seller's works or a monopolist buyer's home to each of two nodal points are equal, this proviso is, of course, satisfied if *any* cost is involved in sales, in the one case from one buyer to another buyer, in the other case from one seller to another

seller. Granted that it is satisfied, it may be to the advantage of a monopolist seller to restrict his sales in the (to him) best market and to dispose of the balance of his produce elsewhere, because, if he acted otherwise, the price offered in his best market would fall and he would lose more by that than he would gain by selling the rest of his product at a higher price than he is actually getting. In like manner, it may be to the advantage of a monopolist buyer to restrict his purchases in the cheapest market and to satisfy the balance of his needs on worse terms elsewhere. This situation is rare with buyers, but common with sellers.

CHAPTER XVI

AN EXCHANGE ECONOMY IN RELATION TO COMPETITION AND MONOPOLY [1]

§ 1. WITH the ground prepared by the three preceding chapters it would be possible, if we so desired, to step straight from a Crusoe economy to one in which voluntary exchange of finished commodities and voluntary co-operation in the production of individual commodities are both present. It is easier, however, to proceed by stages, isolating in the first instance certain aspects of the problem of exchange. For the next section of this inquiry, therefore, occupying Chapters XVI. to XXI., I shall work with a model, which resembles the Crusoe model in that there is no co-operation in the production of any commodity; but many different people are present, making, some one, some another kind of commodity, and exchanging these commodities among themselves. In these chapters no reference is made to capital or land. It is not necessary to the argument to suppose that there *is* no capital or land; but, if there is, each person must be conceived as rigidly connected with and embodying, as it were, inside himself, whatever modicum of capital and land he uses. The issues to be discussed have a much wider reference than is given to them here, but their significance can, I think, be best apprehended when they are displayed in this simplified model.

§ 2. In an exchange economy the product of the marginal unit of any worker's work, the desire for which has to be balanced against his aversion from that marginal unit itself, is not, as it is in a Crusoe economy, the specific addition made by that unit of work to physical product, but a col-

[1] Cf. Appendix IV.

lection of other things obtained by exchange. At first sight this seems to be a point of small importance. Each man is still "producing" the things he wants. He is, indeed, producing them, not directly, but indirectly via a process of exchange; but he is, none the less, in a wide sense producing them. When he exchanges, say a tenth, of the immediate output of his work for so much wheat, he is devoting that much work to securing for himself that much wheat. *Prima facie*, therefore, it would seem that the analysis of Chapter VIII. is applicable to an exchange economy, subject only to verbal adjustments. In truth, however, this is not so. In the more complex economy, when anybody adds an increment to his work, he not only increases his output of service or stuff, but also, because that output is increased, reduces the rate at which it exchanges for other things. Thus suppose that a particular person is making iron in order to exchange it against wheat. What is yielded to him by the marginal unit of his work is the difference made by it to his total receipts of wheat; *i.e.* the quantity of wheat that he obtains in exchange for the additional iron that he is producing *minus* the reduction in the quantity of wheat for which his original output of iron exchanges. The point may be put in more general terms. In an economy consisting of a number of people living together, but not trading with one another, each one will carry his work up to the point at which his aversion from the marginal unit of it is equal to his desire for the marginal product. In an exchange economy, where, as we may suppose for simplicity, service is paid for in money, there are two quantities to be distinguished. First, there is the *value of the marginal product* of a given quantity of our exchanger's work.[1] This is equal to the marginal product of that work multiplied by the price per unit at which the product sells. Secondly, there is the *marginal value of product* of our exchanger's work. This is equal to the difference that is made to his total money receipts by the performance of the marginal unit of his work. It is equal to the value of the marginal

[1] The distinction between marginal private and marginal collective product, which will play a part in later chapters, will, for simplicity, be ignored here.

product *minus* the difference caused in the price of the product multiplied by the quantity of product which he produces. Hence the two quantities coincide provided that the performance of the marginal increment of our exchanger's work has no effect on the price of the product. Otherwise the marginal product of value due to his work is less than the value of the marginal product. There is thus in an exchange economy an element that in a Crusoe economy is necessarily lacking.[1]

§ 3. Now, if an individual producer is selling in two or more markets related in such a way that he is able, in the manner described in the last section of the preceding chapter, to charge different prices at works for different units of his article or service, he can add an increment to his work without affecting the price of the output, certainly of a part, possibly of the whole, of the work he is already doing. In the latter case the value of marginal product and the marginal value of product coincide; and our producer will carry his work unambiguously up to the point at which his aversion from the marginal unit of it is equal to his desire for each of these two identical things. In Chapter XLIII. some study of this situation will be made. Here it is postulated that each producer is constrained to sell at works all units of his output, that emerge at a given time, at the same price. In these conditions an increment of work by any producer is bound to modify the price of the product in *some* degree, except only when he is producing something for exchange against itself, *i.e.* not for exchange at all, but for his own use. It does this partly directly and partly indirectly by causing a somewhat smaller decrement in the output of other producers of the same commodity.[2] If, indeed, any of these other producers were operating under conditions of constant returns, their

[1] What is called here the marginal product of value is called by some writers—*e.g.* Mrs. Robinson in *The Economics of Imperfect Competition*—the marginal revenue.

[2] As the careful reader will observe, the argument of the text tacitly assumes that the effect of one producer's conduct upon the conduct of other producers of the same commodity is determinate. In those conditions, to be discussed in Chapter XVII., where this effect is indeterminate, both the value of marginal product and the marginal product of value to our producer are likewise indeterminate.

decrement and our producer's increment would be exactly equal, so that price would not be affected at all. But, as will appear more fully in Chapter XXIV., in a stationary state this situation is highly improbable, and may safely be left out of account. Hence, except in the special case of production not for exchange, the marginal product of value to any producer of any commodity is bound to fall short of the value of marginal product in some degree. The extent to which it falls short of it will be smaller, the less is the part of the total output of his commodity for which the particular producer is responsible and the more elastic is the general demand for that product. In these circumstances there are two possible objectives open to him: to carry his work so far that his aversion from the marginal unit of it is equal to his desire for the value of the marginal product, or to carry it so far that his aversion is equal to his desire for the marginal product of value. If he chooses the first, we say that he is acting in accordance with the rule of competition. This rule Paréto characterises thus: "L'échangeur subit les prix du marché sans essayer de les modifier de propos délibéré. Ces prix sont modifiés effectivement par son offre ou sa demande, mais c'est à son insu. C'est ce qui charactérise l'état que nous appellons de libre concurrence."[1] On the other hand, if he chooses the second objective, *i.e.* has regard, in deciding on the quantity of his work, to the effect of his action on the price of the commodity, we say that he is acting monopolistically, or exercising monopoly power. These two sorts of action, thus defined, are, of course, all-inclusive. All economic action must either be competitive in character in this sense or monopolistic in character. In an exchange economy, in what circumstances will our producer—we suppose him to be operating in a single market and, therefore, debarred from price discrimination—adopt the one or the other sort of action?

§ 4. To anyone approaching this question from the mathematical side it may seem evident that the maximum principle must always impel him to act monopolistically. For this is

[1] *Cours d'économie politique*, vol. i. p. 20. Cf. Moore, *Quarterly Journal of Economics*, 1906, p. 214.

the logical outcome of that principle. But this is a wrong method of approach. The maximum principle operates upon human beings, not upon electrical machines of perfect sensitivity. The value of marginal product is a thing readily envisaged, and, so to speak, a good natural target. The marginal product of value is much more obscure. If the difference between it and the value of marginal product is small, ordinary people may well fail to recognise that there is a difference. They will then act as though there were in fact no difference. In this case it is the value of marginal product that guides practice; which means that the rule of competition, and not the rule of monopoly, is being obeyed. For the typical producer there is, we must presume, some critical size of reaction upon price, above which he does, and below which he does not, attend to it. What that critical size is cannot be deduced by ratiocination, any more than the degree of physical stimulus just sufficient to be noticed by our senses can be so deduced. It is, so to speak, a piece of brute fact. Whether any individual producer in given conditions will act competitively or monopolistically depends upon that fact in conjunction with the actual difference between the value of his marginal product and his marginal product of value.

§ 5. Up to this point we have tacitly assumed that the separate producers of particular commodities or services act as individuals and do not enter into combinations for the purpose of effecting sales jointly. In fact, however, they may, and often do, do this. Now, when a number of people producing some commodity or service—for simplicity we suppose them all to be alike—agree to regulate their output jointly, the value of the marginal product of any assigned quantity of work is not affected, but the marginal value of the product accruing to each of them is diminished; for now each knows that, if he increases his output by a marginal increment, all the others will do the same, so that the price of the product will be reduced by the effect of their additions as well as of his. The difference between value of marginal product and marginal product of value is, therefore, made larger; the effect on its size varying directly with the pro-

portion of the producers of the commodity or service that are included in the combination. When a large number of these persons is combined together, it is, therefore, much less likely that they will act in accordance with the rule of competition than it would have been had they remained uncombined.

CHAPTER XVII

THE CONDITIONS OF MATHEMATICAL DETERMINATENESS AND INDETERMINATENESS IN A SINGLE MARKET[1]

§ 1. In any single market the price level of all units of commodity dealt in at the nodal point is, by definition, uniform. The purpose of this chapter is to inquire in what circumstances in such a market the desires and productive capacities of the parties are sufficient to determine in the mathematical sense, as defined in Chapter VII., what this price level will be; in what circumstances, in other words, the above data enable us to construct a number of independent equations equal to the number of the unknowns. It will simplify the exposition, without affecting the argument, if we assume that there are no costs of transport inside the market, or, what comes to the same thing, that the whole of it is concentrated at its nodal point. Our inquiry touches on old and famous problems, and the reader must understand that the solutions here given would not win universal acceptance.

§ 2. Let us first suppose that there are only two actual persons, or combined groups of several persons, labelled respectively Robinson and Jones, each producing one sort of commodity or service and bargaining to exchange the whole of it at a single uniform rate against the whole output of the other.[2] Neither holds any stock, but both are engaged in a continuous process of exchange of so much per week, or month, or year, of goods or services that are being continu-

[1] Cf. Appendix V.

[2] The problem is thus *not* that analysed by Marshall in his Note on Barter. In that note he postulates that each of two dealers starts with a certain stock of his own commodity; and he imagines that they engage in a *series* of transactions, which need not be carried out on the same terms.

ously produced. In these conditions the maximum principle yields an equation for the amount of Robinson's output per unit of time, one term of which expresses the way in which shifts in his output will *in his belief* react on the rate of exchange; and it yields an analogous equation for Jones. That is to say, to determine two unknowns, it educes two independent equations, *provided that these beliefs on the part of both Robinson and Jones are given in the conditions of the problem.* In that event the situation is mathematically determinate, no matter what the two beliefs may be. But, where either Robinson's or Jones's belief is not thus given, the situation is mathematically indeterminate until further independent equations have been introduced. Now, if both Robinson and Jones act in accordance with the rule of competition, *i.e.* believe, or, more accurately, act as though they believe,[1] that shifts in their output will not affect the rate of exchange at all, both beliefs obviously *are* given. Hence the situation is determinate. If one of the two acts in accordance with the rule of competition, while the other acts monopolistically, it may seem at first sight that only one of the two beliefs is given, and, therefore, that the situation is indeterminate. In truth, however, if, say, Robinson—it will be remembered that Robinson may be either an individual or a combined group of individuals—acts in accordance with the rule of competition, shifts in Jones's output necessarily affect the rate of exchange in a definite manner. Hence Jones is enabled, by experience of the facts, to form a definite belief about this. Thus his belief, as well as Robinson's, is given; and the situation is determinate. If, however, both Robinson and Jones act monopolistically, neither of their beliefs is given in the conditions of the problem. The view that each of Robinson and Jones takes of the effect on the rate of exchange between his stuff and his correspondent's, that will follow variations in the quantity he offers, depends on

[1] There is, as the reader will notice, a certain awkwardness of exposition here. To avoid the use of the word "belief" in the text would entail much circumlocution. But it must be clearly understood that a mere theoretical belief is of no significance. Belief for our purpose means belief that is acted on. A person who believes in the ordinary sense that a reaction will occur, but disregards that belief in action, for our purpose does *not* believe this.

his opinion as to how the other will respond. But the conditions of the problem do not suffice to determine his opinion on that matter; for how the other will, in fact, respond depends in turn on his opinion of what further response the first will make; and so on indefinitely. While, therefore, we have only two equations, they contain between them four unknowns; and the situation is mathematically indeterminate. Of course, where there are only two dealers, since the difference between value of marginal product and marginal product of value is then bound to be large, both are practically certain to act in accordance with the rule of monopoly. The bargain is thus practically certain to be indeterminate.

§ 3. Let us now suppose that, instead of one Jones confronting one Robinson, there are two Joneses, each making the same sort of commodity. It is implicit in what has already been said that, if any two out of these three persons act in accordance with the rule of competition, while the third acts monopolistically, the situation is determinate. If Robinson and one of the Joneses act monopolistically, while the other Jones acts in accordance with the rule of competition, it is easy to see, on the same lines, that the situation is mathematically indeterminate. *A fortiori* if all three act monopolistically, it is mathematically indeterminate. There remains the case in which Robinson acts in accordance with the rule of competition, but each of the two Joneses acts monopolistically. This is the famous problem of duopoly: for, of course, since we are postulating that Robinson acts on the rule of competition, he is equivalent, for purposes of analysis, to the large free market, in which Cournot and his critics have supposed their rival monopolists to sell. This problem differs from that of a single monopolistic Jones confronting a competitive Robinson in that, while Robinson's obedience to the competitive rule determines what the actual effect on the rate of exchange of a given change in the *sum of the offers of both the Joneses* will be, it does not determine what the effect of a given change in the offer of one of the Joneses will be. Hence, in order for the attitude of each Jones, and so for the whole

situation, to be determinate, some further circumstance must be present, which enforces upon each Jones some definite view as to the effect of a given change in his offer upon the offer of the other Jones. The issue we have to settle is, therefore, whether any circumstance of this character is inherent in the conditions of the problem.

§ 4. Cournot, in developing his mathematics, tacitly assumed that there is such a circumstance;[1] namely, that each Jones regards the effect of a small change in his offer on the offer of the other as nil. On this basis he shows that at every moment each Jones, in order to maximise his satisfaction, will adjust his own output to the then output of the other. When he has done so, his "opponent" will be faced with a new situation and will readjust his output. Successive readjustments along these lines must continue till the two Joneses are producing such outputs that neither of them, the existing output of the other being what it is, could increase his satisfaction by any change in his own output. This position is mathematically determinate; and it is easy to show that the aggregate output of the two Joneses is intermediate between what it would be if they both acted in accordance with the rule of competition and what it would be if they acted in combination as a single monopolist.

§ 5. In contradistinction from Cournot's assumption, another assumption, also implying determinateness, has on occasion been adopted. This is that each Jones, perceiving that there is a certain aggregate output which will maximise the satisfactions of both Joneses together, acts on the belief that, if he contents himself with his proportionate share of this output—with two similar Joneses it will, of course, be one-half—his rival will do the same. For Jones A is bound, it is said, to argue that, though Jones B could gain by raising his output higher than this if Jones A remained passive, Jones B will know that, were he to raise his output, Jones A would not remain passive, but would so act that the final effect on Jones B would be loss and not gain. With this

[1] Cournot, it will be remembered, works out his argument as regards firms seeking to maximise their *money gains*. The essence of his thesis is, however, given here.

assumption, as with Cournot's, the situation is mathematically determinate. In this case the aggregate output of both the Joneses will be the same as it would be, all other things remaining the same, if they were fused together into a single monopoly.[1]

§ 6. If it could be shown that either of the above assumptions about the Jones's views conforms to the facts, it would follow that in actual life the situation under duopoly is mathematically determinate. But this cannot be shown. Consider first Cournot's assumption. The settlement to which it points is the optimum settlement for each Jones, only provided that, if he himself departs from it, the other will not do so. But in fact the other will do so; and everybody knows that perfectly well. We cannot suppose that either will deliberately ignore this knowledge. In other terms, since each Jones knows that the other's output is a function of the price, he cannot without inconsistency believe *both* that, by varying his output, he will cause the price to vary—which is the condition of his acting monopolistically at all—*and* that, by varying his output, he will *not* cause his opponent's output to vary. Cournot's assumption thus not only need not represent the facts: its character is such that—among rational beings—it *cannot* do so. The alternative assumption is more plausible. That it is theoretically possible for each Jones to reason and act in the way it requires, nobody would deny. The question, however, is whether in fact they do so act. Experience in the analogous field of a few competing *firms*—not individuals—strongly suggests that they do not. Even when kartel agreements have been entered into, they frequently break down because some of the members try to steal one another's markets. When there is no formal, or at all events informal agreement, conduct of the type which this assumption postulates has probably *never* been known! I conclude, therefore, that neither of these two assumptions conforms to the facts. Nor, I think, is there any other assumption entailing determinateness for which there is better warrant. The views of the Joneses cannot, therefore, be re-

[1] Cf. Chamberlin, *The Theory of Monopolistic Competition*, chap. iii. and Appendix A.

garded as given in the conditions of the problem; and that problem is, therefore, in general, indeterminate.

§ 7. When the number of Joneses confronting Robinson, whom we still suppose to act on the rule of competition, is increased from two to a larger number, reasoning on the same lines as the above shows that, *provided the Joneses continue to act on the rule of monopoly*, the situation remains indeterminate until their number becomes infinite—or, what is the same thing, until the share that each Jones contributes to the total output becomes infinitesimal. For, so long as any Jones (of given size) himself has regard to the effect of his action on the price-level, he is bound to suppose that other (similar) Joneses will also have regard to it. But, as we saw in Chapter XVI., when the effect to be looked for from any Jones's conduct on the price of the market falls below a certain finite size, that Jones is likely in fact not to have regard to it. That is to say, he is likely to act in accordance, not with the rule of monopoly, but with that of competition. When the Joneses have become so numerous that they all do this—not when their number becomes infinite—the price situation is determinate. How numerous they have got to be depends, as was argued in Chapter XVI., on what effect on the price-level is for them the *minimum sensibile*; and that is a matter of brute fact.

§ 8. In the preceding discussion our single Robinson acting on the rule of competition, has, as was indicated in § 2, been a fiction. A single Robinson would not act on this rule; and what we have, in effect, postulated is a number of them so large that, in fact, they do act on it. Let us now substitute for the fictitious single Robinson a single Robinson who acts monopolistically. Reasoning analogous to that of the foregoing sections then shows that the situation is indeterminate until the number of Joneses has become so large that each of them acts on the rule of competition. It then becomes determinate. If, instead of one Robinson, there are several, it is indeterminate on whatever rule the Joneses act, so long as the Robinsons act on the rule of monopoly. Thus with more than one Jones and more than one Robinson acting monopolistically, the situation is only determinate if both

all the Robinsons and all the Joneses are very numerous. We have determinateness with a sufficient multiplicity in the one group coupled with singleness in the other, or with a sufficient multiplicity in both groups: but not with anything intermediate between singleness and a sufficient multiplicity in either group.

§ 9. So far we have been concerned to distinguish the conditions in which a system is respectively determinate or indeterminate. It remains to add that indeterminate systems need not be unrestrictedly indeterminate. Thus in all the indeterminate systems we have considered indeterminateness is due to the fact that our equations contain as elements beliefs on the part of some Jones or Robinson about the effect on the rate of exchange that will take place if he increases his output; while the conditions of the problem do not enable us to infer what these beliefs are. Even so, however, the data may enable us to find limits within which the beliefs lie. With the help of that knowledge we may be able to demarcate a range within which, in given conditions, the price-level must be situated. This range is generated in a different way, and is of a different character, from the range of *alternative* solutions that is often found in systems that are mathematically determinate; but it is nevertheless a genuine range. To ascertain the extent of it is practically important in connection with wage bargains between workpeople and employers. I have examined that matter at length elsewhere.[1] It is not difficult to see that, as the number of independent bargainers on either side is increased, the range becomes progressively narrower, till, at the stage when the numbers in *both* groups are so large that all members of both act on the role of competition, it becomes a single point.

[1] Cf. *The Economics of Welfare*, Part III. chap. 6, and *The Principles and Methods of Industrial Peace*, Appendix A.

CHAPTER XVIII

MULTIPLE MONOPOLY IN CONNECTED MARKETS

§ 1. In the last chapter attention was confined to a single market. In such a market it was found that, the buyers being supposed to act on the rule of competition, if there is more than one seller acting on the rule of monopoly, the situation will be indeterminate. We have now to inquire what will happen when more monopolistic sellers than one are acting, not in a single market, but in a number of markets. If the markets are separated by distances so wide and costs of transport so large that a change in the price asked for in any one of them has no effect on the quantity of product demanded in any other, they are, in effect, in two worlds. We have then to do, not with multiple monopoly, but with a series of single and separate monopolies. That problem is, of course, determinate. Nobody doubts this, and no discussion of the matter is needed. Suppose, however, that, while sufficient to prevent the markets from being unified, costs of transport are not sufficient to prevent them from interacting with one another. It is this intermediate situation that we have now to study.

§ 2. Two cases have to be distinguished. First, it may happen that, in spite of the existence of costs of transport from the sellers to their buyers, no seller is able to charge different prices at works to different buyers. This condition will be satisfied if any buyer from any one seller, to whom a low price at works is offered, is able to transfer his option to buy to anybody else to whom the seller is attempting to charge a higher price. In these conditions attempts at price discrimination by any seller among his own buyers would

obviously be futile, since no purchase would in fact be made except at the low price. Secondly, it may happen that this type of recontract is somehow barred, so that it is open to any seller, if he wishes to do so, to charge different prices to different buyers. These two cases must be examined separately. Let us begin with the one in which the whole product of every seller must be sold "at works" at a single price.

§ 3. In the problem discussed in the preceding chapter all the units of our commodity were bought and sold at the nodal point of one and the same market. It was, therefore, impossible for rival sellers to be charging at that point different prices; for, if they did so, all the buyers would desert the one whose price was higher in favour of the other.[1] In our present problem each seller is surrounded by a separate market, of which his works constitutes the nodal point. If, starting from equality, one of them sets his price at works above another's, he will lose to him some of his market, but not all; and it may be that the extra price that he secures in the part which he retains more than makes up for the loss. There is no one nodal point at which their prices must be the same, and to that extent the problem has a higher degree of generality.

§ 4. Let us begin by supposing that there are only two sellers, situated some distance apart, among a population of buyers scattered more or less symmetrically over the country, and that the costs of transport of the commodity vary directly with distance. Professor Hotelling in an important article has argued that this situation, on account of "the gradualness in the shifting of customers from one merchant to another", is determinate, though, as he holds, the problem envisaged by Cournot is indeterminate. But his conclusion rests explicitly on the assumption that each seller "adjusts his price so that, with the existing value of the other price, his own profit will be a maximum".[2] This implies the further assumption that each seller believes, and acts on the belief, that, if he alters

[1] I ignore the special case in which the output of one of them is absolutely limited. In that case, if the low-price seller were unable to fill the whole demand for the market at his price, some buyers from the high-price seller would be unable to divert their purchases to him.

[2] "Stability in Competition", *Economic Journal*, March 1929, p. 46.

his own price, his opponent will, nevertheless, not alter his. That assumption is precisely parallel with Cournot's assumption, as described in § 4 of the preceding chapter, viz. that each seller regards the effect of a small change in his offer on the offer of the others as nil. If we accept that assumption, Cournot's case is, I have argued, determinate. In exactly the same way, if we accept Hotelling's assumption, his case is determinate. But, just as, if the reasoning of the last chapter is correct, Cournot's assumption must be rejected, so also must Hotelling's. The problem of two (or more) markets cannot be distinguished from the problem of one market along these lines.

§ 5. That, however, is not the whole story. With two markets, as here contemplated, seller A will know that, if, from any starting point, he lowers his price and B makes no retort, B will not lose customers over the whole of his market, but only in the borderline territory where the two markets are in contact, *i.e.* only among people whose distance from B's works is nearly as great as their distance from A's. If the two markets have a very small spatial range, and if the cost of transport per unit distance is very low, a very small cut in A's price would enable him, should B make no retort, to capture nearly all B's customers. It will be impossible, therefore, for A to believe, or act on belief, that B would in fact make no retort. The Cournot-like assumption which Professor Hotelling makes is patently out of tune with the facts. But, if the markets have a wide spatial range and (or) the costs of transport per unit distance are substantial, a small cut in A's price, should B make no retort, will only enable him to filch from B a minute proportion of his customers. In these conditions he may easily believe, and act on the belief, that B will remain passive; and B may be in like case. The Cournot-like assumption now conforms to the facts; and the situation is determinate. Indeterminateness and determinateness in this problem are thus distinguished from one another by the same class of consideration as decides whether a particular seller will act on the rule of monopoly or on the rule of competition —the brute fact, as described in Chapter XVI., of how strong a stimulus needs to be before people pay attention to it. Thus

for this problem there is no general yes or no solution. Whether a particular situation is determinate or not depends on the details of its setting.

§ 6. In the above discussion we have been supposing that there are only two sellers. If there are a considerable number of sellers, the outlying section of each of their markets will not all impinge upon the market of a single rival, but bits of it will impinge on the markets of each of several rivals. This implies that, provided all the sellers are more or less alike in size, a small shift in the price at works charged by one of them will make a much smaller difference to each of the others severally than it makes to all of them collectively. When this consideration is applied to the analysis of the preceding section, it is easily seen that, with several interconnected markets, the situation is much more likely to be determinate than it is with only two.

§ 7. There remains the second case distinguished in § 2, that in which sellers are free to discriminate in price at works between different buyers. In this case the argument for indeterminateness is much stronger. Let us start with a situation in which hitherto discrimination has been inhibited, and which, subject to that proviso, is determinate. The embargo on discrimination is now removed. Seller A becomes free to reduce his price at works to persons who are buying from B without being compelled at the same time to reduce his price at works to his existing clients. If B does nothing, so long as A keeps the discriminated price above his own costs, he will clearly make an extra profit. B is in a like position. The two thus stand, in respect of territory on the borderline between them, exactly as two sellers, where there are no transport costs, stand in respect of the whole world. In this territory, therefore, the price and output situation is indeterminate. Power to discriminate in price at works among different customers of the same seller thus creates indeterminateness, even though, apart from discrimination, it would not exist.

CHAPTER XIX

MULTILATERAL MONOPOLY

§ 1. Let us now once more leave costs of transport out of account. The analysis of Chapter XVII. was concerned with a market in which all the output of the producers of one commodity is exchanged against all the output of the producers of another. We have seen that in that situation, if each of the two sets of producers (in the same market) is a single person, or single combination of persons, the rate of exchange between the two commodities is indeterminate provided that both exchangers act on the rule of monopoly; and, furthermore, that both exchangers may, in general, be expected so to act. From this it is not unnatural to infer that, if there are a large number of commodities being interchanged against one another, each commodity being the exclusive product of one producer, the resultant situation must also be indeterminate. But this inference is by no means self-evident. It is important to inquire in what circumstances, if any, it is valid.

§ 2. The key to the riddle is to be found in Chapter XVII. § 6. It was there argued that Cournot's fundamental assumption, namely, that each monopolistic Jones acts on the belief that his conduct will not affect the conduct of the other Jones, by means of which he makes duopoly determinate, is untenable because no Jones can be monopolistic unless he reckons that his own conduct does affect prices; and, if he reckons this, he *must* reckon that it will, through price, affect the conduct of the other Joneses. This rebuttal of Cournot's assumption, while unanswerable when trading is

between two commodities only, is not necessarily so when there are a large number of commodities.

§ 3. If two of our traders are engaged in producing commodities that are close substitutes for one another—and the same thing holds good of commodities that are intimately related as complements—the rate of exchange between either of these commodities and outside commodities must clearly be affected in a substantial manner by anything that happens to the output of the other. Hence we are, for the present purpose, in the same position as we should be in if the two commodities were identical. Cournot's assumption fails just as it does in that case, and the situation is indeterminate.

§ 4. But suppose that, of the commodities made by the several traders, none are substitutes or complements in any degree: or, what comes to the same thing, suppose that every set of substitutes or complements is made by the same producer. The above argument now no longer holds. For, when producer A modifies his output, he affects directly the rate of exchange between his commodity and all other commodities. In so doing, he always modifies the rate of exchange of B's commodity with his commodity in the same degree as he affects the inverse rate of exchange of his commodity with B's. But he does not modify the rate of exchange of B's commodity with commodities other than his own in any degree. Hence the proportionate effect on the general value, or price, of B's commodity is not, as it must be if only two commodities are being dealt in, the inverse of the proportionate effect on his own. It is smaller; and, if the number of commodities that are being dealt in is large, it is very much smaller. Hence it is no longer impossible for any producer to take account of the primary effect of his conduct on the price of his commodity and yet not take account of the secondary effect on the price of other producers' commodities, and so upon these producers' conduct. Cournot's assumption, in short, is no longer self-contradictory. On the contrary, with a large number of independent commodities, it is almost certain to be realised.

§ 5. The same point may be put otherwise by contrasting what we may call multilateral monopoly with bilateral

monopoly. The reason why the bargains of bilateral monopoly are indeterminate is that the conditions of the problem do not determine A's belief about the effect on B's offer of a small change in the quantity of his own offer. For a given change in A's offer may cause B's demand curve for his stuff to be shifted, and A has no means of forecasting what the shift in it will be; and similarly for B. When, however, A is selling his stuff to a large number of different monopolists, each producing a different sort of stuff, the part of A's stuff that is exchanged with any one of the others, as also the part of this other's stuff that is exchanged with A, is very small. It is unlikely, therefore, that a shift in A's offer will cause the demand curve—as distinguished from the demand price—of any of the others to be altered; and similarly with B, C and so on. A and everybody else will thus conceive himself as confronting a perfectly definite aggregate demand curve; which means that his output—and so, in like manner, the output of everybody else—will be determinate.

§ 6. Our conclusion then is that, when single producers (or combined groups of producers) of different commodities that are in no degree substitutes or complements are dealing with one another in a system, the smallest possible system, that containing two commodities, is indeterminate; but, as the number of commodities is increased, a critical point is presently reached at which indeterminateness ceases; so that systems containing a large number of different commodities—not being substitutes or complements—are determinate. If there are a large number of such commodities, the sellers of only some of which act monopolistically, the situation is *a fortiori* determinate. If some of the different producers are engaged on commodities that are in some measure substitutes or complements for one another, the relationship not being close or intimate, there is a critical degree of substitutability or complementariness at which indeterminateness passes into determinateness.

CHAPTER XX

THE CONDITIONS OF EQUILIBRIUM FOR AN ASSEMBLY OF ROBINSONS INTERCHANGING AMONG THEMSELVES IN A STATIONARY STATE

§ 1. In Chapter VIII. we studied the conditions of equilibrium for a single Robinson Crusoe producing for himself in a stationary state various sorts of goods and services. In this chapter we are concerned with an assembly of Robinsons, each specialised to a separate commodity—one commodity to each Robinson—trading with one another in a single market with no transport costs, so that the prices of the several commodities in terms of one another are everywhere the same. It is postulated that each Robinson is tied to his own specialism and cannot pass to any other, even though, were he able to do so, he would be substantially better off. The argument of Chapter XIX. has shown that, provided there are a large number of Robinsons, the situation will be mathematically determinate, equally whether all the Robinsons act on the rule of competition, some act on that rule and some on the rule of monopoly, or all act on the rule of monopoly. In each case, with n Robinsons, and so n different commodities to be distributed for consumption among n potential consumers, there are n^2 unknowns and n^2 independent equations. Though, however, all these cases are determinate, they do not, of course, all work out in the same way. It is necessary, therefore, to examine them separately.

§ 2. Let us begin with all-round obedience to the rule of competition. In equilibrium under these conditions, for any Robinson who consumes any part of his own output—but not, of course, for one who consumes none of his own output

—his aversion from the marginal unit of work must be equal to his desire for the product yielded by the marginal unit of work; and also to his desire for that quantity of each commodity against which the output proper to a marginal unit of his work is exchanged, *i.e.* to the value of the marginal product of his work in terms of each commodity. In other words, his desire for the value, in money, of the product of his marginal unit of work must be equal to his aversion from that marginal unit; while the money he receives must be so spent that his desires for what is bought by the marginal units of money devoted to each several sort of purchase are equal. What has been said is true equally whether all the Robinsons operate under conditions of constant returns or whether some Robinsons, having access to free land or free capital that assists them in their work, operate under conditions, not of constant, but of diminishing returns. The latter case differs from the former in that under it they do not get the same rate of pay, whether in terms of their own product or of other products, for intra-marginal units of work as for the marginal unit, but get larger rates of pay. If in these conditions the situations at the several margins and the aggregate amount of work done are the same as they would have been under conditions of all-round constant returns, the aggregate output of commodities must, of course, be larger.

§ 3. If one of the Robinsons acts in accordance with the rule of monopoly, while all the others follow that of competition, the monopolist still devotes to making goods for his own consumption such quantity of work that his aversion from the marginal unit of work is equal to his desire for the output of the marginal unit so devoted. But now the quantity of his work devoted to production for sale—and, with a large number of Robinsons each specialised to different things, this comprises very nearly the whole of his work—is not such that his aversion from the marginal unit of work is equal to his desire for the final proceeds of the marginal unit devoted to production for sale, *i.e.* to his desire for the value of the marginal product of that work. Rather it is so regulated that his aversion from the marginal unit of work is

equal to his desire for the *marginal product of value* due to this marginal unit of work in respect of every commodity for which his own commodity is exchanged. In other words, every Robinson's aversion from his marginal unit of work must be equal to his desire for the product of value in terms of money due to that unit; and, as with all-round competition, the money must be so spent that his desire for what is bought with the marginal shilling is equal in respect of all his purchases.

§ 4. It is evident that the amount of work done by the monopolistic Robinson must be smaller than it would have been had he followed the rule of competition. What of the aggregate amount of work done by all the Robinsons together? It is conceivable that this will be larger when Robinson A acts monopolistically: but circumstances allowing of this are very unlikely to occur. Thus, suppose that the initial situation is one in which Robinson A, like everybody else, follows the rule of competition, and that the product of the marginal unit of his work exchanges against the product of the marginal unit of everybody else's work. Suppose further that conditions of constant return rule everywhere. Robinson A shifts from a competitive to a monopolistic sales policy. It is then easy to show that the amount of work performed by Robinson A, together with the amount performed by the other Robinsons for sale against Robinson A's product, cannot be increased, unless, over a substantial part of the relevant range, the elasticity of demand in terms of other products for Robinson A's product is numerically less than one-half.[1] Moreover, it is to be expected that, if the amount of work done by the other Robinsons for exchange with Robinson A is increased, the amount done by them for interchange among themselves will be in some measure decreased. If this offset is substantial, the elasticity of demand for Robinson A's product must be substantially smaller (numerically) than one-half in order that the aggregate of work done by all the Robinsons together may be increased. Such evidence as there is suggests strongly that elasticities so small as this will seldom be found. For practical purposes, therefore, we may conclude that monopolistic action by Robinson A entails a contraction in the

[1] Cf. Appendix VI. § 2.

amount of work done, not only by him, but by the whole body of Robinsons together. With an elasticity of demand for Robinson A's product over the relevant range numerically larger than unity, other people's work, as well as Robinson A's work, will be cut down. The cut in the aggregate of work done will thus be larger than the cut in Robinson A's work.

§ 5. Passing by intermediate cases, in which more than one, but less than all, of the Robinsons act in accordance with the rule of monopoly, let us consider finally the case in which all of them act in accordance with that rule. Robinson A is supposed to sell to each of the other Robinsons so small a part of his output that no other Robinson, acting by himself, could appreciably modify the price of it in terms of money, and so in terms of his own product, by manipulating the quantity of his purchase. Therefore Robinson A knows that, whether he acts monopolistically or not, his offer of product will be confronted with a demand schedule that is given. This demand schedule is not rendered indeterminate by uncertainty as to the way in which other Robinsons will react to changes in the amount of Robinson A's offer; but is determinate. So far the situation is the same whether Robinson A alone acts monopolistically or whether all the Robinsons act monopolistically; but, of course, if they all act monopolistically, the total output of work is different from what it would have been had only one of them done so. The effect on this total output is equal to the sum of the effects that would have been produced on it by each Robinson turning separately to monopoly while all the others continued to act in accordance with the rule of competition. It is conceivable that under all-round monopoly the aggregate of work done would be larger than under all-round competition; but, as with a single monopolist, the conditions which would permit of this happening are very unlikely to be realised. Practically speaking, it is certain that, in the orbit of our present problem,[1] all-round monopolisation will be associated with an aggregate volume of work substantially smaller than would be forthcoming under all-round competition.

[1] For a related, but different problem, cf. *post*, Chapter XLI.

CHAPTER XXI

A COMPARISON OF TWO STATIONARY STATES OF AN ASSEMBLY OF ROBINSONS INTERCHANGING COMMODITIES UNDER THE RULE OF COMPETITION[1]

§ 1. THE discussion of the preceding chapter was an extension of that developed in Chapter VIII We have now in a similar manner to extend to an assembly of Robinsons the analysis of Chapter IX. That is to say, we have to study, not the equilibrium conditions of a single stationary state, but the relation between two stationary states proper to two sets of conditions that differ in certain specified respects. In order to make this problem manageable I shall confine myself to the case in which (1) every Robinson acts in accordance with the rule of competition and (2) the work of every Robinson is performed under conditions of constant return. I shall study the difference between two stationary states which are alike in every respect save that in one of them (*a*) a particular Robinson's technical capacity to produce his commodity has been improved and (*b*) the collective desire of all the Robinsons for a particular commodity has been augmented. Attention is confined to improvements in productivity of a sort that are equiproportional for all relevant quantities of commodity and to increases of desire that are similarly equiproportional.

§ 2. The first part of this chapter is concerned with an assembly of two persons only, Robinson and Jones, the more complex problem of large assemblies being postponed till the final section. Each of Robinson and Jones is specialised to the production of one commodity and they interchange in a single market, with nil transport costs, all or some of their

[1] Cf. Appendix VII.

respective outputs. The postulate that they act in accordance with the rule of competition must in this case be made, so to speak, by fiat. Each of them is regarded as a "representative particular". This is necessary; for in actual life, with only two exchangers, the rule of competition would not, as has already been observed in Chapter XVII. § 2, in fact be followed. I shall investigate in turn two cases: (i) that in which each of the traders consumes one of the two kinds of product only and (ii) that in which each of them consumes some of both products.

§ 3. Given that Robinson and Jones each consumes one kind of product only, two situations are possible. On the one hand, each may consume the commodity that he himself produces. In this case there is no trade, and each constitutes a separate one-commodity Crusoe economy. On the other hand, each may consume the commodity which the other produces. In this case there is the maximum possible amount of trade. The difference between the two situations can be set out most simply thus. In each of them there are two commodities A and B, A being produced by Robinson and B by Jones. In each situation there are two desire schedules of given constitution, tied respectively to commodity A and commodity B. The two situations differ in that in the Crusoe economy the desire schedule for A is wielded by the producer of A and that for B by the producer of B, while in the exchange economy the desire schedule for A is wielded by the producer of B, and conversely.

§ 4. Let us begin by studying directly, in an exchange economy, the effect (i) of an improvement in productivity in respect of Robinson's commodity and (ii) an increase in Jones's desire for Robinson's commodity, on the aggregate quantity of work done. With an improvement it is obvious that in this kind of economy, as in a Crusoe economy, if the elasticity of marginal desire for the commodity whose productivity has been improved is equal (numerically) to unity, the improvement will cause no change in the quantity of work done by either Robinson or Jones. If the elasticity of Jones's marginal desire for Robinson's product is numerically greater or less than unity—the elasticities of their

marginal aversions from work not being nil—the quantity of work done by each of them, and so, in general, the quantity of work done by both together, will be affected. With an increase in Jones's desire for Robinson's product these quantities—apart again from the case of nil elasticities of marginal aversion from work—will be affected in all circumstances. Provided that the elasticity of Robinson's marginal desire for Jones's commodity and of Jones's marginal desire for Robinson's commodity are both numerically greater than unity, it is plain to common sense that an improvement and an increase in desire will both cause the aggregate quantity of work done by Robinson and Jones together to increase. It is plain too that in both cases the increase will be larger, the more elastic are each of the two marginal aversions from work, and the more elastic numerically are each of the two marginal desires. If one or both of the two elasticities of marginal desire are numerically less than unity, common sense is doubtful. In Appendix VII. it is shown that the doubt is justified. Alike with an improvement and with an increase of desire, unless the two elasticities are *both* numerically greater than unity, the aggregate quantity of work done may, according to the detailed conditions, either increase or decrease. The ways in which, in the two cases, the size of the increase or decrease is related to the size of the elasticities of marginal aversion from work and marginal desire for product, are very complex. In the Appendix they are determined by algebraic analysis, and the results are summarised in a table of signs.

§ 5. So far we have been considering an exchange economy by itself. Let us now compare it with a Crusoe economy, in which the desire for each commodity is wielded by the producer, not of the other, but of that commodity. With an improvement in Robinson's productivity, if Jones's marginal desire for Robinson's commodity has an elasticity numerically equal to unity, the quantity of work done is not affected at all in either economy, *i.e.* is affected equally in both. But, apart from this case, we should expect *prima facie* improvements and increases of desire alike, in conditions under which they cause an increase in aggregate quantity of work,

to cause a larger increase in an exchange economy than they do in a Crusoe economy. For in an exchange economy the quantity of work done by two persons, whereas in a Crusoe economy the quantity done by one only, is disturbed. But this is not decisive. For it may be that in a Crusoe economy the effect on the quantity of work done on the commodity primarily affected is larger than in the other. The problem is studied, with the help of algebraic analysis, in Appendix VII. It is there shown that, provided the elasticities of marginal aversion from work of the producers of the two commodities are equal, and also the elasticity of marginal desire for the one commodity is equal to that for the other, the effect of an improvement on the aggregate quantity of work done is exactly the same in both economies. In the case of an increase in desire for one of the commodities, if the above provisos are satisfied, the effect is again, just as it is with an improvement, exactly the same in both economies. If these provisos are not satisfied, the effects alike of an improvement (except when the elasticity of marginal desire for the commodity in respect of which it is made is numerically equal to unity) and of an increase in desire are different in an exchange economy from what they are in a Crusoe economy.

§ 6. Let us next allow each of Robinson and Jones to consume some of his own product. There now come into the picture four new elements: (1) the elasticity of Robinson's marginal desire for his own product; (2) the proportion of Robinson's work that is devoted to providing that product for his own consumption; (3) the elasticity of Jones's marginal desire for his own product; and (4) the proportion of Jones's work that is devoted to providing that product for his own consumption. These new elements greatly complicate the problem. For example, with an improvement in Robinson's productivity, the more elastic is Robinson's marginal desire for his own product, the larger will be the increase in work done by him. But at the same time the more difficult it will be for Jones to secure more of Robinson's product. Hence, if Jones's desire for Robinson's product is elastic, the smaller will be the increase in work done by him. By parity of reason-

ing, the larger is the proportion of his work that Robinson devotes to providing his product for his own consumption, the larger will be the increase in the quantity of work done by him, and the smaller, if Jones's desire for Robinson's product is elastic, the increase in the quantity done by Jones. In like manner, the smaller is (1) the elasticity of Jones's marginal desire for his own product and (2) the proportion of his work that he devotes for providing this product for himself, the larger will be the increase in the quantity of work done by him, and the smaller the increase in the quantity done by Robinson. In view of these complications—and analogous complications are associated with changes in Jones's desire for Robinson's product—it is not surprising that only meagre conclusions of a general kind can be reached. It is still true that, if all the elasticities of marginal desire are numerically greater than unity, both an improvement in productivity and an increase in desire in respect of either commodity must cause the aggregate quantity of work done by Robinson and Jones together to increase. But the conditions on which the magnitude of the increase depends are so complex that it is not worth while to set them out even in symbols. There is, however, one simple and sweeping conclusion yielded by mathematical analysis. Provided that the elasticities of marginal aversion from work of Robinson and Jones are equal and also that all four elasticities of marginal desire are equal, then, alike with an improvement and with an increase in desire, the change induced in the aggregate quantity of work done in the complex economy that we are now studying is exactly the same as that induced, with the same proviso about elasticities, in an economy where Robinson and Jones interchange the whole of their outputs; which change, as we saw in § 5, is itself the same as that induced in a Crusoe economy where there is no trade at all. This is true whatever the values of the elasticities may be, and whatever the proportion of their outputs that Robinson and Jones retain for their own consumption.

§ 7. So far we have supposed that there are only two traders. Let us now envisage a large assembly of persons, each specialised, as before, to producing one commodity, and

each consuming only one commodity. Now, however, nobody necessarily purchases the commodity he consumes from the person by whom his own output is purchased for consumption. Thus, let the traders be A, B, C . . . Z. A may consume the output of someone who does not consume A's output. Any number of degrees of round-aboutness or many-corneredness are possible. If A sells his product for consumption to B, who sells his for consumption to C, who sells his for consumption to A, there is three-cornered trade. If A sells for consumption to B, B for consumption to C, C for consumption to D, D for consumption to A, there is four-cornered trade. In general, with n links in the chain, there is n-cornered trade. When everyone consumes his own product, $n = 1$, and we have, if we will, one-cornered trade; with precise balancing of imports and exports to each correspondent two-cornered trade. Evidently an improvement in the productivity of any person or an increase in the desire of any person for product cannot affect the quantity of work done outside the group to which he is linked. Hence we have a clear-cut issue: to determine in what way the magnitude of the effect on the aggregate quantity of work done, when this effect is not nil, is related to the size of n. Mathematical analysis shows—a result *prima facie* surprising—that, if the elasticities of marginal desire of all the traders for the commodities they severally consume are equal, and if the elasticities of their marginal aversions from work are also equal, the change induced in the aggregate amount of work done, alike by an improvement in productivity and by an increase in desire for any product, will be the same whatever the number of links the group contains. Many-corneredness of trade *as such* has no tendency either to aggravate or to mitigate this change. Granted that each trader consumes one kind of commodity only, the aggregate quantity of work done is affected to exactly the same extent in a Crusoe economy—one-cornered trade—a bilateral exchange economy—two-cornered trade—and a multi-lateral exchange economy. When the several elasticities of marginal desire for commodity and of marginal aversion from work that are involved are not all equal, this is, of course, no longer true. In that

more complex case no general conclusion of reasonable simplicity can be established, save only—what is indeed obvious—that, with all the elasticities of marginal desire numerically greater than unity, an improvement in productivity and an enhancement of desire in respect of any commodity will both cause the aggregate quantity of work done to increase.[1]

§ 8. There remains the most complicated case of all, that of an assembly of many Robinsons trading with one another, each still specialised to producing a single kind of commodity, but now consuming, instead of one commodity only, a large number of different commodities. For an assembly of three, or indeed, any finite number of Robinsons, the consequences of an improvement in productivity, or of an increase in desire of all the Robinsons together in respect of one commodity, could, with sufficient labour, be expressed in algebraic formulae. But these formulae would be a mere chaos of symbols, yielding no information of general interest, save only, as above, that, if all the elasticities of marginal desire are numerically greater than unity, an improvement and an enhancement of desire will both cause the aggregate quantity of work done to increase. We may imagine, however, if we will, a perfectly symmetrical system, where every Robinson is doing the same amount of work and consuming the same proportion of the output of each sort of product, and where all the elasticities of marginal desire for commodity are the same, as also are all the elasticities of marginal aversion from work. In this case yet once more the effect of an improvement, and equally the effect of an increase in desire for any commodity, on the total quantity of work done is exactly the same as it would be in that form of Crusoe economy in which everybody consumes one kind of commodity only.

[1] A part of the argument of chap. v. of Part I. of my *Industrial Fluctuations* needs to be revised in the light of what has been said in this section.

CHAPTER XXII

THE ECONOMIC CHARACTERISTICS OF A ONE-COMMODITY COMMUNITY

§ 1. In Chapter XIII. it was observed that the actual world differs from a Crusoe world in two respects. Different finished commodities and services are exchanged against one another, and different people co-operate in the production of the same finished commodity. In the last six chapters we have been considering a model in which the first of these two characteristics of the actual world is taken into account, but not the second. I now proceed to a third model in which the first characteristic is not taken into account, but the second is. For this model there are no direct personal services. The only consumable good is a single species of economic cake, all the parts of which are exactly alike, and which, the state being stationary, is produced, shared out and eaten in the same manner and in the same quantities year after year. There is, in short, co-operation in production, but no exchange of resultant finished objects. The co-operators are, not merely persons, but the whole body of factors of production, human and non-human together. As was observed in Chapter V., these may be grouped in the classical manner under the broad heads labour, or more properly labour power, capital and land. But each group must be conceived as divided up into a number of sub-factors, mental labour power of various grades and kinds, manual labour power similarly subdivided, innumerable different kinds of capital objects, and innumerable pieces of land of diverse chemical quality with diverse climatic conditions. Individual items belonging to one or another sub-factor may wear out and dis-

appear. But, since our state is stationary, when this happens, they are always exactly replaced, and the total stock of every factor and sub-factor is maintained intact. In these conditions the annual output of cake will be of a certain size, and the distribution of it among the various factors and sub-factors, through whose co-operation it is produced, will be effected in a certain manner. The twin problems of production and distribution are thus presented *minus* the associated problem of exchange, and, therefore, in a highly simplified form. The business of the next thirteen chapters is to study a model of the economic world constructed on this plan.

§ 2. As a prelude it is convenient at this stage to clear out of the way a *prima facie* difficulty. For the purpose of our analysis it will be necessary to think of the joint product of the various factors of production as being distributed among them through payments made to each at such and such a rate of pay for use of a physical unit in each production period. In common life, however, the rate of pay to capital instruments is conceived as such and such a rate of interest upon the cost of making them. We thus have labour paid at so much per man, land at so much per acre, both physical units, but capital paid at so much per unit of value. The relation between this popular usage and the usage of our analysis needs to be made clear. In truth it is perfectly straightforward. As has already been implied in Chapter X., the two usages are alternative ways of expressing the same thing; the translation from the one form into the other is mere book-keeping. If we were able to estimate the cost of production of a workman of given skill, whatever wage he earned could, if we wished, be set down as so much per cent on this cost. In actual life, when a man has bought a farm, he sometimes speaks of his income from it as so much per acre and sometimes as such and such a percentage on the purchase price. If we had records of the money cost of the whole existing stock of capital equipment, we could express the rate of pay per unit of it as a percentage of that cost. In fact, we have not these records; and the percentage we actually set out is always that which the rate of pay per unit makes with the current cost of *new* units. Applying the

percentage thus obtained to the income derived from old-standing capital equipment, we can, if we will, calculate the present value of that stock, which must, in a stationary state, be equal to the original cost price of it. In brief, in given conditions the rate of pay per year for the use of a physical unit of equipment *is* so much. This so much is such and such a percentage of the cost of making now a new unit of the equipment, *i.e.* allowance made for a depreciation, it *is* such and such a "rate of interest" upon that cost. Our *prima facie* difficulty thus disappears.

§ 3. Experience has shown that there are advantages in dividing the influences we have to study into two groups, operating respectively, in Marshall's language, on the side of demand and the side of supply. In what follows I shall make use of this dichotomy. There is, however, more than one way in which this might be done, and it is necessary to make precise the procedure that I shall adopt. What is directly demanded is the flow of services rendered by the stocks of factors of production. If the mere existence of a unit of any factor, labour power, capital or land, entailed the provision by it of such and such a fixed rate of service per year, demand for the services of a stock of factors and demand for the maintenance of the stock itself would be identical. It is to be expected, however, that in actual life different rates of pay will evoke different flows of service from *given* stocks of factors. There are thus two variable elements involved, not one only. So far as *a priori* considerations go, the element, rate of flow of service per unit of stock, may be entered equally well on the demand side or on the supply side of the account. That is to say, we may equally well place the demand for such and such a flow of service against the supply of that flow, in which case our element enters into supply; or we may place the demand for the use of such and such a quantity of stock against the supply of that quantity—in which case the element enters into demand. The choice is purely a matter of convenience. In what follows I shall put this element on the supply side. On the demand side I shall study the price offered for various quantities, not of factors, but of service rendered by factors. The relation between the

quantity of a factor and the quantity of service rendered by it will come in on the other side.

§ 4. The concept, quantity of service rendered by a factor, does not, however, enter easily into the analysis that has to be undertaken. For that we need the concept quantity of factor at work. The step to this can be taken without great difficulty. Thus suppose that only one degree of intensity of work is possible, so that the services rendered by a unit of any factor vary proportionately with the length of hours per day during which it is in action. If we then select an arbitrary standard length of day, say eight hours, the quantity of factor at work is obtained by summing the number of hours worked by each unit of the factor and dividing by eight. Obviously the quantity of factor at work so obtained stands in all circumstances in the same proportion to the quantity of service rendered by the factor, and so can be used in place of it. Of course in actual life intensity of work as well as length of working day is liable to vary. This circumstance can, however, be brought into account in the manner indicated in Chapter VIII. § 3, and does not prevent us from expressing quantities of service rendered by a factor in terms of quantity of factor at work.

§ 5. In economic text-books the term supply is used indifferently for rates of flow of commodity or service and for quantities of standing stock. This use of the same word in widely different connections easily leads to confusion. There is a special danger of this in discussions about capital. When the supply price of so much capital is spoken of, it is not always clear whether the price that will call out such and such a rate of new investment is intended, or the price that will maintain in being such and such a stock of capital equipment. The term supply suggests an inflow of something, not the mere maintenance intact of a fixed stock; and, therefore, even when it is the latter that is meant, the reader's mind is pushed towards the former. It would, of course, be possible to obviate confusion by always speaking of supply price with an explanatory clause. But the supply price of a stock of something is an unnatural and perplexing collocation of words. The logical solution would be to speak of the

supply price of flows, but of the *maintenance price* of stocks, whether in the aggregate or as devoted to a given purpose. With this usage such and such a price at once *maintains* such and such a stock of a factor in existence and evokes from that stock such and such an annual *supply* of service. If it were adopted, we should have to speak of the supply price of the services rendered by a factor, *i.e.*, in the language of the last section, of the quantities of the factor at work, and to set this against the demand price. I propose, however, for reasons of convenience to use for this purpose the term maintenance price. The maintenance price of any assigned stock is, the reader should note, necessarily identical with the supply price of a nil rate of net addition to (or subtraction from) that stock.

§ 6. During the next twelve chapters we shall be concerned with a one-commodity community, where all the units of every factor—for convenience I use the term so as to include sub-factors—of production are in a single market, as defined in Chapter XV., between the different parts of which all of them can move without cost or hindrance, so that the rate of payment of every unit of each factor is the same everywhere. Chapters XXIII.-XXVII. are directed primarily to influences on the side of demand, Chapters XXVIII.-XXXII. to influences on the side of maintenance, and Chapters XXXIII.-XXXIV. to the interactions of the two groups of influences. The consequences that follow when the assumption of free costless movement is withdrawn are reserved for separate study in Chapter XXXV.

CHAPTER XXIII

MARGINAL PRIVATE AND MARGINAL COLLECTIVE PRODUCT OF FACTOR GROUPS AND OF INDIVIDUAL FACTORS OF PRODUCTION

§ 1. In earlier chapters the concept marginal product has been employed in reference to an isolated Robinson Crusoe. It has been defined, in respect of any given quantity of work done by him, as the difference made to his total output by a marginal increment of work after enough has been set aside to make good any damage incidentally done to his capital stock and his natural resources. We have now to extend this concept, first to Robinsons who do not function in isolation, or, more generally, to linked groups of factors bound together in fixed proportions—I shall call them factor groups—which are not isolated; and secondly, to individual factors of production which are not isolated.

§ 2. In the case of factor groups, there is no difficulty about the allowance for making good damage done to the owner's stock of capital or natural resources. This plays exactly the same part as it does with an isolated Robinson. If, of the gross output of the marginal unit, say, one-quarter goes to making good wear and tear, the net product, or, in our usage, the product, of the unit is three-quarters of the gross output, and cannot possibly be anything else. Yet again, as with an isolated Robinson, no question can arise about there *being* such a thing as a marginal product. With the general assumption of free divisibility into small units, as described in Chapter XI., there obviously *must* be such a thing. We may, therefore, pass at once to our problem without any preliminary clearing of the ground.

§ 3. If there is only one centre of production, the marginal product of any number of units of a factor group is simply the difference made to total (net) product by the marginal unit. The term is thus wholly unambiguous. When, however, there are several centres of production, this is not so. The marginal unit of each factor group is more exactly conceived, not as all concentrated upon a single centre, but as distributed in the optimum manner over all of them. Thus, with n (similar) centres, it is made up of n parts, each amounting to $1/n$th of a unit, standing in each of the several centres. Each of these n parts must yield a product in the centre where it is standing, but it *may* also yield a product (positive or negative) in the other centres. We have, therefore, to distinguish between (1) the yield of each part in the centre where it is standing and (2) its total yield. It is no longer possible to use the term marginal product without qualification. I shall, therefore, call the sum of the yields of the several n parts in the centres where they are respectively standing the *marginal private product*, and the sum of the total yields of the several n parts the *marginal collective product*. The former of these is approximately equal to the addition to the product of the centre where it is engaged that results from increasing the quantity of factor-group units there by one unit, whether that unit is introduced from outside or is withdrawn from one of the other centres. The latter is approximately equal to the addition to the total product that results from this when the unit is introduced from outside. It is, therefore, legitimate to use the terms marginal private product and marginal collective product in these senses as well as in the more exact senses distinguished above. In what follows I shall do this. Whether we do it or not, the essential point to bear in mind is that, while the marginal private product and the marginal collective product of any given number of factor group units *may* coincide, there is no *a priori* necessity for them to do so.

§ 4. Common sense suggests—and for purposes of a rough approximation the suggestion is adequate—that coincidence between the two sorts of marginal product implies, and is, indeed, another name for, absence of what Marshall has

called external economies and of external diseconomies; and divergence the presence either of external economies or of external diseconomies. But this identification is not quite exact. Conditions are possible in which an increase in the scale of output, *i.e.* in the quantity of factors at work, indirectly brings it about that a different size of centre becomes the optimum; and in this new optimum size the marginal private product of the factor group units may be larger or (conceivably) smaller than it was in the old. This situation does not lend itself to description in terms of external economies or diseconomies, and has led some writers to add to this familiar category the new hybrid, external-internal economies.

§ 5. A word should be added here in caution against a possible misapprehension. It might be thought at first sight that a distinction analogous to that drawn in § 3 holds good between the average product of a single centre and average product on the whole. This is not so. Any excess that there may be in the collective over the private product of any unit of work done in one centre necessarily appears in the private product of the work done in other centres; and, in like manner, any shortfall in the collective, as against the private, product of work done in one centre necessarily so appears. It follows that, while marginal collective and marginal private product may differ, average collective and average private product must always be identical.

§ 6. Let us turn to individual factors of production. Here some preliminary clearing of the ground is needed. First, whereas, as we have seen, for factor groups, as for isolated Robinsons, the question whether there is such a thing as marginal product is idle, for individual factors this is not so. Situations are possible in which no difference is made to output by a change in the quantity of any of the several factors unless the quantities of all of them are changed in the same proportion. Thus it may be that in the art of brigandage nothing whatever could be accomplished by a man alone or a horse alone or a rifle alone, but only by a man equipped with one horse and one rifle; or—a more complex case—that, in order to accomplish anything, each quantity of one factor must be associated with specified quantities of each of the

other factors, different proportions being needed in respect of different quantities of the one factor. In conditions of this kind the marginal productivity of the individual factors would be quite indeterminate. Marginal product in any significant sense would not exist. But these conditions imply a discontinuous function. The functions with which we have to do are continuous, or, more strictly, approach near enough to continuity to allow of the rough edges being disregarded without serious error.[1] In these circumstances variations in the quantity of any one factor, the quantities of all the others being held constant, affect output by finite amounts. That is to say, they have a marginal product, whose magnitude depends, for each of them, the quantities of the other factors being given, on its own quantity.

§ 7. Secondly, there is a difficulty about the allocation of the incidental damage caused by productive activity to the stock of producing factors. In so far as the rate at which capital and exhaustible natural resources, such as coal deposits, wear out or are used up, is higher, the greater is the quantity of labour that is co-operating with them—and to some extent this can hardly fail to be so—not all of the costs of this wear and tear should be deducted from the gross product of the marginal unit of capital or "land" when the net product—or product—of that unit is being calculated; a part of these costs should be set against the marginal product of labour. If we had to determine how much should be so set, the problem would evidently be a serious one. In the practice of business men, however, it is common to regard annual capital depreciation as depending simply on the size of the capital stock: such and such a type of machine has such and such a normal length of life, *i.e.* depreciates at such and such a rate per annum. The depreciation rate reckoned for coal-mines and so on also rests on a convention; while for the stock of labour no depreciation rate is reckoned.[2] We can hardly do other here than follow these conventions. Adopting them, we conclude that, for the factor labour, gross and net marginal product are identical, while for capital and "land" marginal net falls short of marginal gross

[1] Cf. *ante*, Chapter XI.

[2] Cf. *ante*, Chapter V. § 6.

product by the depreciation rate that is customarily set against them.

§ 8. We are now in a position to consider, in relation to individual factors of production—when there are more factors than one—the distinction, that was drawn above for factor groups, between marginal private and marginal collective product. From the general principles of analysis we know that, with a continuous function of more than one variable, the increment to the total value of this function that results from increasing together the several variables by small amounts, is equal to the sum of the increments that would result from increasing by the same amounts each of these variables separately. Hence—for simplicity we may suppose units so chosen that the number of units of each sort of factor in the factor group are the same—the sum of the marginal products, whether private or collective, of the individual factors is equal to the marginal product, whether private or collective, of the factor group. This relation is fundamental. It does not follow, however, that the ratio of marginal private to marginal collective product is the same for each of the several factors as it is for the factor group in which they are contained.

CHAPTER XXIV

THE DIVISION OF FACTOR-GROUP UNITS AMONG CENTRES

§ 1. As we premised in Chapter I., the fundamental driving force in economic life is the maximum principle as it was defined in that chapter. In any given state of industrial knowledge, aversion attitudes towards work and desire attitudes towards products, this principle determines what concrete arrangements will be made. In a one-commodity community, between the different parts of which movement for factors of production is perfectly free, so that the same rates of pay prevail everywhere, one essential feature of these arrangements stands out clearly. The quantities of all the factors being given, should they be divided up among separate centres in discrepant proportions, output could always be increased, and so gain made, provided that every factor is divisible into sufficiently small parts, by shuffling them about until they stand in the same proportion in all the centres. This follows from the general rule, of which more will be said presently, that an increase anywhere in the quantity of one factor of production, while the quantities of the other are unchanged, entails diminishing returns. Hence the maximum principle requires that the several factors shall be bound together in joint units—"factor-group units"—each of which embraces the several factors in proportions corresponding to their several stocks. These factor-group units may be made up, according to the number of factors in existence, of one sort of factor only or of several. But in either event any one factor-group unit—whether its constitution is simple or complex—is exactly like any other. The task of the present chapter is to inquire in what manner

the maximum principle will regulate the number of these units embraced in the several centres of production, *i.e.* the size, and so the number, of the said centres.[1]

§ 2. If the owner of each factor-group unit were completely isolated without any interaction whatever between him and others, each of them would be a Robinson Crusoe confined—on our present model—to the production of a single commodity: and there would be nothing further to be said. I postulate, however, that the owners are free, if they will, to co-operate in their work, producing jointly an output which accrues, not to each owner in his own place, but in a homogeneous mass, that has to be distributed by agreement among those who have made it. In these conditions the maximum principle will evidently lead to co-operation when co-operation enables a given effort to produce a larger output than it could produce without it. It will entail combination into groups of that size beyond which an increase yields no addition to average, and so to total, output.

§ 3. Conditions are conceivable in which average output per factor-group unit would be the same whatever the size of the separate combinations in which they are linked. The distribution of factor-group units into centres of production would then be completely indeterminate. Centres of any size would be equally possible, and there would be no reason for different centres to be of the same size. Some writers incline to the view that these conditions are realised; that the scale of output is affected by the way in which different factors are combined in the factor-group units, but not by the way in which factor-group units of given constitution are combined together. On this basis they suggest that the advantage which large-scale centres of production sometimes have over small is due merely to some of the elementary factors being imperfectly divisible. Thus Professor Ohlin writes: "If all the productive factors—raw materials, tools

[1] In order to keep the argument reasonably simple it is assumed in the text that each centre of production constitutes a single business, arrangements under which several centres are united for some or all purposes in the same business being ignored. If these arrangements were brought into account, the detailed analysis required would be more complicated, but its general principles would not be affected.

and implements, were completely divisible . . . the most economical combination . . . would be equally possible on a small scale or on a large".[1] As the context shows, however, Professor Ohlin is not here using the term divisible to mean physically divisible, or even physically divisible without cost or effort, but with some reference to efficiency. For him, so long as a centre composed of factor-group units is of less than the most economical size, some factor embraced in the centre is *by definition* not perfectly divisible: he does not deny that larger structural units may be more efficient than smaller ones. This is all that we need. Indeed, it is evident that the all-round size of a centre often makes a large difference to productivity per factor-group unit. Thus suppose that the factor-group embraces one kind of factor only, namely, manual labour, and that the "commodity" to be produced is moving a block of iron weighing ten tons along a certain route. Unless the labour gang undertaking this operation surpasses a certain size, it will be unable to "produce" anything at all. No doubt, if we imagine, not merely the factors of production, but the whole surrounding environment to be divisible into small pieces without cost or effort—so that our ton of iron is in powder—this would not be so. But such a conception is fantastic. In conditions in any degree resembling those that actually exist complete physical divisibility of the factors of production would not render the size of the centres into which they are combined a matter of indifference. The absolute quantity, and not merely the proportionate quantity, of the factors embodied in a centre are relevant to its efficiency, so to speak, per unit weight. There is nothing strange about this. On the contrary, it is in accord with the general facts of biology. Absolute size, no less than structural form, is relevant to an animal's fitness to deal successfully with its environment. Because heat loss is proportionate to surface area and not to cubic contents, very small animals cannot exist in cold climates. In like manner, for purely technical reasons, the most efficient airship, *i.e.* the one that will travel at a given speed with least horse-power per ton, is very large; while, *per contra*,

[1] *Interregional and International Trade*, p. 53.

the most efficient aeroplane is comparatively small. Thus, I conclude, average productivity per factor-group unit is, in general, different for groups of different sizes, and there is, in general, some optimum size. It is conceivable, indeed, that there should be several different sizes in respect of which average productivity is the optimum: and, indeed, it is probable that in any given industry for different forms of organisation, *e.g.* the private firm form and the joint-stock form, there will be different optimum sizes. We may, however, reasonably rule that case out of account and confine ourselves to situations in which there is only one optimum size. It is, of course, assumed that all the processes entailed in the manufacture of a unit of our imaginary one commodity are conducted together in the same centres. If they were separated, the optimum sizes of the centres engaged in different processes would, of course, in general, be different.

§ 4. What this optimum size is may be partly dependent on the total number of factor-group units available for employment in all the productive centres collectively. It will not be rendered thus dependent merely by the presence of external economies. In order that it shall be so, the range of *internal* economies must be affected by the aggregate scale on which factor-group units are employed.[1] That condition may or may not be satisfied. Whether it is satisfied or not, the dominant influences determining optimum size are those that govern the relation between internal economies and the scale of individual businesses. A very important place among them is held by the system, or technique, of organisation, by means of which the work of a co-operating group is harmoniously arranged. That system corresponds in industry to the apparatus by which, in living things, oxygen is supplied to the cells and carbon dioxide removed. It is impossible for

[1] Thus, if x be the number of factor-group units co-operating at a given centre and y the number at the sum of the other centres, the output of the centre may be expressible as $F(x)+\phi(y)$. In this case the optimum size of the centre is given when $\frac{d}{dx}\left\{\frac{F(x)}{x}\right\}=0$, and is thus independent of y. On the other hand, the output of the centre may be expressible only in the more general form $F(x, y)$. In this case its optimum size is given when $\frac{d}{dx}\left\{\frac{F(x, y)}{x}\right\}=0$; and is not independent of y.

creatures with the structure of insects to be large, because their apparatus of branching air tubes entails a slow diffusing process, that can only work effectively provided the tubes are short. For vertebrate animals, on the other hand, oxygen and carbon dioxide are transported by a rapidly circulating blood stream: they are, therefore, capable of attaining much greater size.[1]

§ 5. What has been said so far concerns the real ultimate conditions governing optimum size. These real conditions act, as it were, *through* a more immediately visible formal condition, the nature of which is well known. When the quantity of factor-group units is given, and it is a question of distributing them among productive centres, marginal *collective* product—the difference to *total* output that results from increasing the total quantity of factor-group units—obviously has no relevance, since this quantity is not being increased. But it is easy to show that, all the factor-group units being supposed exactly alike, the optimum size of a productive centre will be that size in respect of which the marginal *private* product is equal to the average product of the factor-group units engaged there. For, if marginal private product is larger than average product, average product can be increased by increasing the size of the productive centres. In the converse case it can be increased by diminishing the size of the productive centres. The formal condition that must be satisfied for any centre to be of optimum size is thus perfectly clear-cut.

§ 6. Before we proceed to the next stage of the argument it is necessary here to interject a word of caution. When the separate factors have determinate marginal private products,

[1] So far as pure theory goes, co-ordination does not necessarily imply the setting apart of a particular person to do the work of co-ordination. A cricket team, for example, might conceivably share the work of captaincy by consultation in a committee where everybody's vote counted equally. In practice, of course, a particular person will be appointed for this work. At first sight this might seem to be inconsistent with our concept of a system of co-operating factor-group units all exactly alike. There is not, however, any real inconsistency. For precise similarity in the quality of the individual units does not entail that they are all doing precisely similar jobs. A cricket team *must* arrange itself in the field in different positions, one at point, one at mid-off, and one at square-leg; and this equally whether the men have different capacities or are all identical twins!

we might be inclined to extend to them the conclusions reached in the last section about factor-groups. If we did this, we should conclude that that optimum size of centres, which makes the average product and the marginal private product of the factor-group at each centre equal, also makes the average product and the marginal private product of each several factor equal. Now it is true, as was shown in the last chapter, that the marginal private product of the factor-group in any centre is necessarily equal—units being so chosen as to make the quantities of all the factors the same—to the sum of the marginal private products of the several factors. Hence the sum of these marginal private products is equal to the average product of the factor-group. But this does not entail that the marginal private product of each individual factor is equal to its average product. That need not, and indeed, in general, cannot happen. For an individual factor average product, if it means anything at all, can only mean the integral of its marginal private products divided by the quantity of the factor: just as, in like manner, average collective product can only mean the integral of the marginal collective products so divided. But, in general, for individual factors of production the familiar law of diminishing returns holds good. It may well be, indeed, that for very small outputs it does not hold. For, if the minimum unit of one kind of factor is large, less than a certain number of units of the other factors may be unable to co-operate with it effectively. With substantial quantities of output, however, experience suggests that it always does hold. The quantities of all other factors being given, the more of any one factor there is, the smaller its marginal product, whether private or collective, will be. It follows that, while, for optimum distribution among centres, the sum of the marginal private products of the several factors and the average product of the aggregated factor-group engaged must be equal everywhere, the marginal private products and the average products of the several factors individually are not equal.

§ 7. This being understood, we may revert to the main theme. The optimum size, in the sense defined in § 5, of a productive centre being determined in the way we have

described, it seems at first sight that the maximum principle will cause the existing stock of factor-group units to combine in centres of this size, thereby making average and aggregate output as large as possible. This *prima facie* view is, however, encountered by another diametrically opposed to it. According to that view in its crudest form each centre of production must necessarily be ruled by a "business man", and all business men, since they are free to start businesses as they choose, must necessarily be employed. If this is so, the number of centres is governed by the available number of business men, and the actual size has no tendency to coincide with the optimum size. Now, if business men—in the sense of people who act as heads of businesses—were a race apart, there would be force in this contention. But, of course, business men in this sense are not a race apart. They are particular members of a wide group, capable of acting either as heads of businesses or in the office of manager or other type of organiser. The number of those who rule productive centres, and, therefore, the number of these centres, is thus not determined *ab initio* by an outside consideration. It is itself dependent upon what size of centre is the optimum. The larger the optimum size of centres is, the fewer members of this wide group will act as heads of businesses, and the more in other capacities: and *vice versa*. This objection to our *prima facie* view, therefore, breaks down.

§ 8. It does not follow, however, that that view is precisely correct. If it so happens that the extant number of factor-group units is an exact multiple of the number needed to constitute a centre of optimum size, then, in a thoroughgoing stationary state, it is, indeed, correct.[1] But that condition may fail in either of two ways. On the one hand, the extant number of factor-group units may be larger than the number required for a centre of optimum size, but not an

[1] In the form of stationary state displayed in Chapter II. § 4 it is not correct. For there individual firms are not for the most part in equilibrium, but are passing through different stages of growth and decay. They would differ in size, therefore, at any given moment, just as, in a population, infants, boys and men differ in size, even though the sizes proper to each of them, and to which each of them at maturity attains, are the same.

exact multiple of that number; on the other hand, it may be smaller than the number required for a centre of optimum size.[1] In both cases it is impossible for all the extant factor-group units to be combined in centres of optimum size; and our problem is still incompletely solved.

§ 9. When the extant number of factor-group units is larger than the number required for a centre of optimum size, either of two arrangements is *prima facie* possible. Suppose that the number of units required for such a centre is k, and the number of the extant units $(kn + l)$, where l is less than k. Then *prima facie* there may be constituted n centres of optimum size and one centre containing l units; or there may be constituted either n centres, each containing $\frac{kn+l}{n}$ units, or $(n+1)$ centres each containing $\frac{kn+l}{n+1}$, according as l is $<$ or $>$ $\frac{1}{2}k$. Knowledge that in the actual world there are in most industries productive centres of various sizes may perhaps suggest that the former alternative is the more probable. When, however, we reflect on the conditions postulated for a stationary state, we see, not merely that this is not so, but that that alternative is impossible. For, if there were one centre of less than optimum size, it would pay the units attached to it either to distribute themselves among the other centres or to induce some members of those centres to combine with them in their centre, till, in one way or another, all the centres were of equal size, and one or other of the two forms of the second alternative were realised. Thus all the centres will be somewhat larger or somewhat smaller than the optimum size: none of them will conform exactly to that size.

§ 10. When the extant quantity of factor-group units is larger than, but not an exact multiple of, the number required for an optimum centre, the above arrangement maximises aggregate output, subject to that condition. If the situation is such that n is small, the size of the actual centres may be much larger or much smaller than that of the optimum centre. But, if n is large, if, that is to say, the centres are numerous, actual size and optimum size are very nearly the

[1] The possibility that the optimum size might be smaller than one factor-group unit may safely be left out of account.

same, and, for practical purposes, we may say that all the centres *are* of optimum size.

§ 11. When the extant number of factor-group units is smaller than the number required for a centre of optimum size, there will obviously be established one centre of production only, in which all these units are assembled. Under this arrangement aggregate and average output will again be larger than it could be under any other arrangement. But the condition, equality between marginal private product (which, where there is only one centre, is, of course, the same as marginal collective product) and of average product will be inhibited, and, it may be, a wide divergence will be created, by *force majeure*.

§ 12. In the cases considered in the three preceding sections the economic cosmos is in one sense out of equilibrium. For the arrangement towards which the maximum principle is pressing is not attained. An obstacle has been encountered —the unsuitable number of extant factor-group units— which it cannot overcome. In another more usual sense the system is in equilibrium in spite of this fact; just as a book lying on a table is in equilibrium in spite of the failure, engendered by that obstacle, of its tendency to fall to the ground. This, of course, is purely a verbal matter.

CHAPTER XXV

THE CONSEQUENCES OF QUALITATIVE DIFFERENCES AMONG PIECES OF LAND AND UNITS OF ORGANISING POWER

§ 1. At this stage it is well to make explicit a very important respect in which our analysis misrepresents the facts. Our purpose being what it is, the model that we study is rightly constructed on a pattern enormously less complex than the reality whose outline it is designed to imitate. Some falsification is, therefore, inevitable. But, unless the general character of the falsification is clearly understood, darkness rather than light will be generated, and careless readers will assign to reality attributes which by no means belong to it. Hence this cautionary interlude.

§ 2. Underlying the argument of the preceding chapter, as it will also underlie the main body of later analysis, is the assumption that there are in play a reasonably small number of factors and sub-factors of production, each of which is homogeneous, in the sense that every unit of it is of like quality. In actual life there are for certain factors, notably land and some kinds of brain power, large differences of quality between individual units. Now some quality differences are expressible, by a simple device, in terms of quantity. Thus we can imagine two kinds of land differing in such a way that 2 acres of the one kind are exactly equivalent in all respects to 1 acre of the other. So much of other factors of production applied to $2n$ acres of this kind yield exactly the same total, and also exactly the same marginal, product, as an equal quantity applied to n acres of the other kind. In this case we can, if we wish, choose our units of land, not by area, but in such a way that two acres of the first kind con-

tain the same number of units as one acre of the second kind. When we have done this, in respect of units so chosen everything will be true that would have been true with acres as units had all land been of identical quality. Quality differences of this type present no difficulty. But many quality differences are not thus simple. There must, of course, be some acreage of Class B land in conjunction with which any given quantity of other factors will yield the same total product as it would yield in conjunction with one acre of Class A land; and also some acreage in conjunction with which it will yield the same marginal product as it would yield in conjunction with one acre of Class A land. But the quantities of Class B land required respectively for equal total products and for equal marginal products may be different. Moreover, the differences themselves may be different, whether reckoned absolutely or as proportions, according as the quantities of the other factors that are being employed are large or small. When pieces of land, and, of course, the same thing is true of any other productive agent, vary in quality in this complex fashion, it is impossible by any device to express quality differences in terms of quantity.

§ 3. The implications of this situation as regards the size of centres of production are most easily exhibited if we suppose the factors other than land to be perfectly homogeneous, but land itself to consist of several grades of divergent quality. The grading must not—what has just been said precludes that—be conceived as something absolute without reference to the quantities of the other factors that are available. When, however, the quantities of the other factors are given, a reasonably satisfactory definition can be constructed. I say then that one piece of land is better than another if, these other factors being distributed in the optimum manner, it will pay to assign more of them to the first piece. The other factors then will so spread themselves over the various grades of land that their marginal private products, as applied to every acre of every grade to which any of them is applied, are equal. It may be that there is enough first-grade land to absorb all the other factors, *i.e.* that any applied to second-grade land

would yield a smaller marginal private product than is yielded by all applied to first-grade land. In this case the other factors will assemble exclusively upon first-grade land; and the situation will be exactly the same as it would have been had the other grades of land not existed. If there is not enough first-grade land available, some units of the other factors will settle upon and combine with second-grade land. These units will yield the same marginal private product and receive the same rate of remuneration as other units do. But the proportions in which the other factors are joined with the land (measured in acres) will be smaller for second-grade land than for first-grade land. If there is not enough of first- and second-grade land, some third-grade land also will be utilised, with similar consequences; and so on. Thus, in general, the proportions of land and other factors in the centres of production will be different with different grades of land. Inside the aggregate mass of factors associated with each grade centres of production will be separated off on the principles indicated in the preceding chapter; every centre being of the same size. But the centres settled on different grades of land, not being similarly constituted, cannot be equal in size. Even though they should contain equal quantities of all the other factors and equal acreages of land, they would not in any strict sense be of the same size, since land would mean different things for different centres. But in fact the quantities of the other factors will be larger in centres containing the better grade land than in those containing worse grade land.[1]

§ 4. Very similar considerations come into play when

[1] In general, we may expect the rate of pay per acre to be larger for better grade than for worse grade land. But, with our definition, it *need* not be larger. For better grade land only means land that would yield a given marginal private product for a larger quantity of the other factors than the quantity to which worse land would yield it. If the better grade land yields nearly constant returns and the worse grade sharply diminishing returns to early doses of the other factors, rent per acre, and even gross produce per acre, *may* be larger on "worse" land. (Cf. Marshall, *Principles*, Book iv. Chapter 3). It will be observed that, in speaking of a larger quantity of the other factors, we are tacitly making the unreal assumption that the other factors are everywhere combined together in the same proportions.

account is taken of differences of quality among the various units of organising ability that are engaged in production. Here too the differences of quality are too complex to be resolved into differences of quantity. An able man is not simply the equivalent of two or more stupid men. He is a different kind of man, and no number of stupid men combined together could exactly take his place. In centres controlled by men of high organising power larger quantities of the other factors will be grouped together than in centres controlled by men of less power. Since there are not likely to be many men of the highest grade of organising capacity, we may expect that many inferior grades also will have to be invoked for the task of captaining centres of production. Hence these centres will not in fact be of one size only, but of various sizes spread out in a more or less normal frequency distribution. Those organisers whose organising power is too small for them to have any clients *qua* organisers are not, however, left derelict as waste land is, but will continue to work in some capacity other than that of heads of businesses or controlling managers.

§ 5. It is not proposed in this book to follow further the lines of inquiry that these considerations suggest. If quality differences among the units of our factors were *sufficiently* widespread—if, to take an extreme case, every unit of every factor were unique—the method of analysis which we have been adopting, and which is common to nearly all economists, would break down. The mathematical concept of a function would be inappropriate. Though quality differences among our units are, no doubt, important, they are not important enough for that. By leaving them out of account, as, in the main, I am doing here, we do not, I think, render the broad principles of our analysis inapplicable to real conditions.

CHAPTER XXVI

POSSIBILITIES OF INDETERMINATENESS WHEN THE ACTUAL AND THE OPTIMUM SIZE OF CENTRES OF PRODUCTION DIVERGE

§ 1. Let us now return to the main channel of the argument. When the actual and the optimum size of centres of production coincide, the average product of factor-group units and the marginal private product are, as we have seen, equal. The demand price per factor-group unit is then obviously equal to both these quantities.[1] When, however, the actual size and the optimum size of productive centres diverge, average product and marginal private product are different. The demand price—rate of pay—cannot then be equal to both of them. If factor groups consisted of one sort of factor of production only, this would not lead to any doubt about determinateness. Provided that there is a uniform rate of pay —and competition among the several units of the factor may be supposed to ensure that—the demand price *must* be equal to average product, marginal private product becomes irrelevant, and there is nothing more to be said. But in real life factor-group units consist, not of one, but of many sorts of factors. As we have already seen, appropriate units being

[1] To avoid complicating the argument unduly I have tacitly assumed in the text that there is no interval between the application of the agents of production to cake making and the emergence of finished cake. Of course, in fact there is such an interval. Consequently, the real demand price of any agent is not its actual marginal private, *i.e.* average, product, but this discounted in respect of that interval. There is, however, now in play a new agent of production, namely the stock of working capital in the form of cake in process. Payment to this agent absorbs the annual balance of cake, that, in consequence of the discounting, remains over after the other agents have been paid.

chosen, the marginal private product of any given quantity of factor-group units is equal to the sum of the marginal private products of the several factors they contain. In the conditions supposed it is impossible for the demand prices of the extant quantities of these factors to be equal to their marginal private products. Nor can we now say that these prices are equal to their several average products. Indeed, as we saw in Chapter XXIV. § 6, the notion of an average product of one among a number of co-operating factors of production is either meaningless or obviously irrelevant. These negative results, while not suggesting doubt about the determinacy of aggregate output, inevitably do suggest it about the determinacy of the rates of pay to the several individual factors. This doubt we have now to examine.

§ 2. Let us suppose first that each factor individually is engaged and paid in each centre by a representative of all the factors collectively. As each factor comes separately to the door, the representative will unhesitatingly pay out to each the equivalent of its marginal private product—until the last factor comes. At that point, if the actual size of the centres is larger than the optimum size, he will find that he has too little product to make a payment at that rate; if the actual size is smaller than the optimum size, he will find that he has too much. Thus every factor will be paid at a rate equal to its marginal private product except the one that comes for payment last; and this one will be paid what is left over. If then our data do not tell us which factor will come for payment last, the sharing of the product is, on the data, indeterminate; if the data do tell us this, it is determinate. In actual life, in general, some factor, through its agents, hires the others. The hiring factor is not necessarily the same as the organising factor. Thus a group of manual workers or of capital owners may hire managers. But some one factor is unambiguously the hiring factor. Clearly, when this is so, that factor, whatever it is, is the one that comes for payment last. We are, therefore, entitled to say that, in the conditions here contemplated, all the factors other than the hiring factor are paid at a rate equal to their marginal private product and the hiring factor is paid what is left. It follows that the shar-

ing out of the product is, on the data, determinate or indeterminate according as these data do or do not include information as to which factor is acting as the hiring factor. If we take the standpoint of extreme abstraction, our data do not tell us that; but, if we step nearer to reality and inform ourselves by observation whether capitalists through joint stock companies, organisers through private businesses, or manual workers through workmen's copartnership associations are in fact acting as hirers, our data are extended; and on those data the situation is determinate.

§ 3. Whether the above analysis has any practical significance depends, of course, on whether centres of production are in fact of optimum size. For, if they are, the question what would happen if they were not has only academic interest. As will appear presently, in a many-commodity community centres are sometimes pushed away from optimum size because they adopt the rule of monopoly rather than that of competition. In a one-commodity community, where it is only possible for a centre to sell its output against other units of the very same commodity, value of marginal product and marginal product of value always coincide. This implies that monopolistic policy and competitive policy lead to identical results.[1] Divergence of actual from optimum size cannot, therefore, be brought about by monopoly. The only way in which it can arise is through the aggregate extant quantity of factor-group units not being an exact multiple of the quantity needed to constitute a centre of optimum size. We have seen, however, that, whether it is an exact multiple or not, if the former quantity is very large compared with the latter, the actual size of centres will, for all practical purposes, coincide with the optimum size. Now, in a one-commodity community, in which there are no costs of transport to split it into parts, regarded as representative of actual communities, the quantity of factors available in the one (imaginary) industry must be much greater than the quantity available in actual communities for any actual industry. In view, therefore, of the fact that in many actual industries we find a large number of separate

[1] Cf. *ante*, Chapter XVI. § 3.

centres, it is reasonable to suppose that in our model one-commodity community there must be an enormous number. This entails approximate identity of actual and optimum size. No doubt, if we had to imagine the one commodity produced through the agency of several different *kinds* of firms—as cotton is, in effect, produced partly by spinning-mills and partly by makers of textile machinery, and coal partly by coal-miners and partly by makers of dynamite—the warrant for it would be weakened. A single dynamite firm can make enough dynamite to serve an enormous number of coal-mines; so that among dynamite firms actual and optimum size might easily diverge. But our model one-commodity community may fairly be pictured as consisting exclusively of firms of one kind only. In that case there is no divergence. As yet, therefore, the class of situation contemplated in § 2 does not call for study.

CHAPTER XXVII

PRODUCTIVITY FUNCTIONS AND DEMAND PRICES OF FACTORS IN A ONE-COMMODITY COMMUNITY WHERE ALL THE CENTRES OF PRODUCTION ARE OF OPTIMUM SIZE

§ 1. IN a one-commodity community the demand side of the economic shield is dominated by the general productivity function. This function expresses, in respect of any given set of quantities of the several factors of production, the total output of all the factors together. In the economics of real life a study of the influences which make the separate productivity functions of the principal industries what they are has an important place. Plainly, however, this kind of study cannot be usefully conducted in respect of an imaginary cake economy. Here we must be content to assert in broad terms that the general productivity function is an outgrowth of the state of industrial knowledge, the attainments of science, the degree to which relevant knowledge is disseminated —not held secret by particular groups of producers—and the development of methods for applying it in appropriate industrial organisation. In this catalogue I have deliberately set down the state of industrial knowledge, not, as perhaps might have been expected, the state of industrial technique. The reason is that technique denotes the nature of the operations by which industry is actually carried on; and this depends, not merely on the state of industrial knowledge, but also on the way in which groups of industrial agents are made up. With a given state of knowledge, for example, it will be different if there are a small number of roughly specialised types of factor from what it is if there are a large number of finely specialised types. Thus technique is itself

a function of industrial knowledge *and* of the quantities of the several factors. It is proper, therefore, to go behind it. No doubt, in the last resort, even the state of industrial knowledge is in part dependent on the quantities of the factors, different sorts of knowledge being developed with different systems of factors. But it is not necessary for our purpose to bring into account this ultimate complication.

§ 2. At first sight the reader may be puzzled as to the relation between the general productivity function and the arrangement of factor-group units among centres of production. Here we appear to be saying that output depends on the form the general productivity function without reference to that arrangement, whereas in Chapter XXIV. we argued that that arrangement affects output in an important way. The explanation is that the general productivity function, though it does not refer explicitly to distribution among centres, does refer to it implicitly. Given the state of industrial knowledge and so on, together with the quantity of the factors, these things make output what it is by ensuring that distribution of the factors, which, given all the facts, conforms to the maximum principle. This point, which may perhaps seem somewhat obscure when set out in words, is obvious in the symbolism of the accompanying footnote.[1]

§ 3. It is sometimes supposed that, while the general conditions of productivity, as expressed in the productivity function, govern the aggregate amount of output, a different set of conditions governs the way in which output is shared among the factors, so that production and distribution constitute, as it were, independent branches of economics. This is not so. When the quantities of the several factors are given, the conditions that lie behind the general productivity function govern both the quantity of the aggregate output and also—provided that every unit of each factor is paid at the same rate—the real receipts of the several factors. This broad statement must serve as a starting-point. But it does

[1] Thus, if P be annual output, x, y, z, quantities of factors and n number of (similar) centres, we may either write $P = F(x, y, z)$, or we may write $P = \phi(x, y, z, n)$, subject to the condition that $\partial\phi/\partial n = 0$. With the function F, this condition, which is made explicit in the function ϕ, is implicit.

not fully represent the facts, and needs to be supplemented in an important particular.

§ 4. We have seen in the preceding chapter that, if the aggregate number of factor-group units is large relatively to the number required to constitute a centre of optimum size —a proviso that we may properly assume to be satisfied in a one-commodity community with nil transport costs for factors of production—all the centres will in fact be approximately of optimum size. This implies that the marginal private product of factor-group units is everywhere equal to the average product. But the marginal private product of a factor-group unit is equal to the sum of the marginal private products of the individual factors contained in it. Hence, if the individual factors are paid at rates equal to their marginal private products, the whole product will be precisely absorbed. There is thus no internal inconsistency in the thesis that the demand prices of the several factors are equal to their respective marginal private products. This state of things is possible. But, provided it is possible, there is no difficulty in showing that it is also necessary. For it is worth while for the other factors assembled at any centre to offer for an increment of any one factor any rate of pay not greater than its marginal private product; and, all the units of that factor being exactly alike, competition with other centres prevents them from actually offering less than the most that it is worth their while to offer. It follows that the demand price in each centre for the amount of each factor assembled there is equal to its marginal private product there. Since, moreover, in the conditions here supposed every factor distributes itself among the centres in such wise as to make its marginal private product the same in all of them, this entails that the demand price for each factor as a whole is equal to its marginal private product as a whole. This conclusion holds good equally whether or not monopoly action is attempted by anyone controlling the whole or a part of the stock of any factor; and, if monopoly action *is* attempted, equally whether it is one-sided or two-sided. Such action can in no case affect the demand price for any factor otherwise than by modifying the quantity of

that, or the quantity of other factors, that is at work. These quantities being modified, the marginal private product of each factor is, in general, modified; but, whatever happens to it, it and demand price always and necessarily coincide.

§ 5. There can be no question, on the assumptions that we are here making, that this analysis rightly depicts the facts as regards factors of production that are hired. But in every centre there must be, besides hired factors, also some one who does the hiring. There is no known mechanism—no glorified totaliser—that will take over this person's work. Sometimes, indeed, the hirer is himself also hired. The manager of a joint-stock company, who, directly or through subordinates, hires employees, is himself hired, *via* the directors, by the stockholders. Again, the manager of a workers' copartnership association may be hired by the workers collectively, while they themselves are hired by him individually; just as the captain of a cricket team may be elected by potential members of the team, with a free hand afterwards to select the team. But, besides hirers of this kind, there are many hirers, the heads of private businesses, not themselves hired by anyone. With regard to these hirers there is a difficulty. Since they are not directly engaged by somebody else, there is nobody to balance at the margin what is paid to them against what they contribute to output. We cannot, therefore, prove that the demand price for their services is equal to their marginal private product in the way that we can prove this about the other factors.

§ 6. There is, however, another way of proof. As we saw in Chapter XXIV., units being so chosen as to make the quantities of all the factors equal, the marginal private product of the whole factor group is equal to the sum of the marginal private products of the several factors. But, provided that all the centres of production are of optimum size—and we have agreed that in a one-commodity community, with nil transport costs for factors of production, this may be taken to be so—the marginal private product of the factor group is equal to the average product. If then all the other factors are paid at rates equal to their individual marginal private products, what is left for the remaining member of the group,

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viz. the factor that acts as hirer, necessarily constitutes a rate equal to its marginal private product. At first sight, indeed, this argument seems open to a serious objection. In a normally constituted centre of production there is only one hirer, or, at all events, only one supreme hirer. How can we speak of the marginal private, or any other sort of marginal product of a single indivisible person? The answer is that, as has already been indicated in Chapter XXIV. § 7, the hirer does not constitute in himself a separate factor of production, whose function is simply hiring. The function of hiring is attached to a particular one of a group of people of given quality, the group qualified, for example, to act indifferently either as heads of businesses or as hired managers. It is this group of persons, not the single hirer, to which marginal productivity is attributed; and it is this group the demand price for which is proved by the foregoing argument to be equal to its marginal private product. The ultimate implication of that argument is that the particular member of this group, to whom the office of hiring is assigned, is not paid at any different rate, because this function is assigned to him, from what his compeers are paid in other offices, or from what he himself would be paid if employed in another office. The seat of the hirer is not a privileged seat, occupancy of which carries special payment. Hiring is a particular job, like accounting or interpreting, to which certain men of a particular grade are posted; but those who are posted to it are not paid more than the others, any more than those who are posted as accountants or interpreters are paid more than the others. When this is understood, the *prima facie* objection to our argument dissolves, and that argument, subject, of course, to the condition that all the centres of production are of optimum size, passes unscathed through the fire.

§ 7. But we have still to bring into account the famous "adding-up difficulty". This difficulty, which has long troubled mathematically-minded economists, is not disposed of by postulating that all the centres of production are of optimum size. Until recently the now familiar distinction between marginal private and marginal collective product was not

taken; and the difficulty was set out thus. By a natural development of what was said in earlier sections it was shown that the demand price for every factor of production is equal to its marginal product *sans* phrase; this marginal product of each factor being measured by the partial differential, in respect of that factor, of the general productivity function. But, unless the productivity function is homogeneous in the first degree, unless, in other words, it is of such a sort that an equal proportionate change in the quantities of all the factors entails the same proportionate change in the quantity of output, unless, in yet other words, conditions are such that no economies or diseconomies would result from an increase in the scale of output, payment in accordance with these demand prices will not exhaust the product. It will either absorb less or—*per impossible*—more than the whole of it. Since there is nowhere else for product to go and nowhere whence a deficiency can be made good, this result appears to imply that the productivity function *must* be of that type. But it is impossible that a proposition about the nature of reality, whether it be true or not, should be demonstrable in this *a priori* manner. Hence we are involved in a logical impasse, and the whole principle of marginal analysis is infected with doubt.

§ 8. So soon as the distinction between marginal collective and marginal private product is taken, a way is opened up for disposing of this difficulty. Economies and diseconomies due to an enlargement of the scale of output—ordinarily spoken of as economies of large scale—may be available either because the centres of production are of more or less than optimum size, or because, these centres being of optimum size, the marginal collective product of factor-group units differs from the marginal private product. Now at this stage of the inquiry we are postulating that the centres *are* of optimum size. Hence, for the general productivity function not to be homogeneous in the first degree, or, in other words, for economies or diseconomies of large scale to be available, *means* that the marginal collective product of factor groups differs from the marginal private product; which means, in turn, that the sum of the marginal collective products of

the individual factors differs from the sum of the marginal private products. Now it is only the marginal collective products, not the marginal private products, of the several factors, that are measured by the partial differentials, in respect of them, of the general productivity function. But the demand price of every factor, in the conditions here supposed, is equal to its marginal private product. It is the remuneration of all factors in accordance with their demand prices which must exactly exhaust the product. This does not entail the productivity function being homogeneous in the first degree. It *may*, of course, be thus homogeneous, but there is no logical compulsion that it should be. If it is not, payment of all the factors in accordance with their marginal collective products would not precisely exhaust the total product. But there will be no tendency for them to be paid in accordance with their marginal collective products. There is thus no impasse.

§ 9. Upon this analysis there follow certain obvious corollaries. If marginal private product is equal to marginal collective product, marginal private product is unchanged by a change of output. For a change of output means simply a change in the number of centres, all of them of optimum size and in full work. If marginal private product is greater than marginal collective product, this means that there are diseconomies of large scale: marginal private product, therefore, decreases as output increases. In converse conditions, there are economies of large scale, and marginal private product increases with increases of output. In other words, equality of marginal private product and marginal collective product entails constant returns; excess of marginal private over marginal collective product diminishing returns; excess of marginal collective over marginal private product increasing returns.[1] Of course, the returns here spoken of are physical returns. What has been said must not be taken to imply that, in a particular industry, in a many-commodity

[1] It will be understood that constant returns here mean constant returns from a macroscopic standpoint. If we were to take a microscope and examine the consequences of *sufficiently* small variations of output, the curve of marginal productivity would appear, not as an absolutely straight, but as a slightly wavy, horizontal line.

community, equality of marginal private and marginal collective product carries with it constant supply price. For that we *also* need that (i) the proportions in which, in the particular industry, the several factors of production are engaged and (ii) their several money prices are unaffected by the scale of output. The remote possibility that constant supply price may be established through increases in the scale of output entailing, on the one hand, economies of large scale and, on the other, increases in factor costs which precisely offset these, may be ignored.

§ 10. It remains to inquire whether, in a one-commodity community not broken up by costs of transport, divergence between marginal private and marginal collective product is in fact likely to occur, or whether we may reasonably postulate, not only that the actual and the optimum size of centres of production coincide, but also that there is no divergence between these two sorts of product. Now, the larger the quantity of factor-group units that is available, the more likely it is that the full possibilities of external and external-internal, as well as of internal, economies are exhausted. But a quantity of factors, which would warrant the postulate that the actual and the optimum size of centres of production coincide, will not warrant this postulate. To make it plausible a much larger quantity is needed. Nevertheless, I suggest that, for a model one-commodity community, in which there are no costs of transport, it may reasonably be adopted: and in the equilibrium analysis of Chapters XXXIII.-XXXIV. I shall adopt it, as well as the other postulate. Both postulates together entail, of course, that there are no economies or diseconomies of large scale.

CHAPTER XXVIII

THE GENERAL CHARACTERISTICS OF MAINTENANCE PRICES

§ 1. In our analysis of demand the stocks of factors at work, about which discussion centred, were, of course, stocks at work for some specific number of hours per day. From the standpoint of productivity the stocks at work would be *pro tanto* "smaller" if they worked shorter hours. Our terms must, of course, be used with the same significance in maintenance analysis as in demand analysis. Throughout that analysis, therefore, I shall mean by the stock of any factor or sub-factor at work, not the physical stock of it at work, but this stock corrected by an index that measures the rate of flow of work or activity from it. Thus, if the higher of two prices entails a 20 per cent larger quantity in physical stock associated with a 5 per cent smaller number of hours of work (of given intensity) performed by it per day, the stock, in my sense, is larger, not by 20 per cent, but by 15 per cent.

§ 2. The maintenance price of a given stock of factor or sub-factor at work is the price which, *in the surrounding conditions*, will maintain that stock constantly at work. Thus, if p be the maintenance price of a stock x, it is the supply price of a nil rate of flow of additions to the stock: for labour of a nil rate of increase or decrease in the working population; for capital of a nil rate of investment or disinvestment—or what, when we are speaking in real, as distinct from money terms, is, on all definitions, the same thing—of saving or dis-saving.

§ 3. In a one-commodity community it is obvious that the demand conditions for the several factors and sub-factors cannot be independent, but must constitute an inter-con-

nected system. That is to say, the demand price of a given quantity of one factor or sub-factor depends in part on how large stocks of other factors or sub-factors are at work. On the maintenance side, however, the existence of a system is less self-evident; and, indeed, it is not uncommon for discussions of the maintenance conditions of, say, labour in general and capital in general, to be kept in entirely separate compartments. Such a way of approach is misleading. Maintenance conditions, as well as demand conditions, constitute a system. An exhaustive discussion of this very complicated matter is beyond my present scope. I shall content myself with exhibiting in the next chapter the inter-connections on the maintenance side of different kinds of labour power, and in the one that follows the inter-connection of labour in general and capital in general. These two chapters will be followed by two others, in which the relations between rates of pay and the stocks at work respectively of labour in general and capital in general are studied separately subject to the condition that other things are equal.

CHAPTER XXIX

THE INTERRELATIONS OF THE MAINTENANCE CONDITIONS OF DIFFERENT KINDS OF LABOUR

§ 1. In Chapter V. a distinction was drawn between finished sub-factors of production and certain more ultimate factors, which we may regard, if we will, as raw material capable of being transformed, with or without cost, into any one of a number of different sub-factors. The situation is further complicated by the fact that certain sub-factors can be derived indifferently either out of one type of raw material or out of another; *e.g.* telephone operators can be manufactured at need either from male persons or from female persons. In the present chapter I shall provisionally suppose the quantities of the factors—the ultimate material—to be fixed, and shall concentrate attention upon the interrelations, on the maintenance side, of the several sub-factors. As was stated at the end of Chapter XXVIII., I am concerned in this chapter only with labour.

§ 2. The fundamental governing principle has already come faintly into our picture, as it were out of due time. In postulating nil cost of transport between places, we have already, for convenience of exposition, brought into account on the demand side an element that, in strictness, belongs to the maintenance side. For, when in these circumstances the total quantity of any finished sub-factor of production is given, the reason for price equality in all the centres where it is engaged lies in the conditions of maintenance. The quantity that is maintained in any one centre will be nil if the price is less than the price elsewhere, and will exhaust the whole stock if it is greater than the price elsewhere. The

distribution of ultimate factors among sub-factors is governed from the maintenance side by the same broad principle as the distribution of sub-factors among centres of production; though the application of this principle is more complex. The particular types of productive agent into which people of given inborn quality convert themselves are determined by a comparison of the real prices offered for different types. The quantity of each type that is maintained is related, not to one price, but to a system of prices. Our present business is, the system of prices and the stock of ultimate factors being given, to exhibit the way in which the quantities of the several sub-factors are determined.

§ 3. Let us first focus attention on a set of sub-factors, the whole of every one of which is made out of one single kind of ultimate factor. Thus the relevant ultimate factor consists of a number of men, all of them initially exactly alike in general make-up, having exactly the same *comparative capacity* for different sorts of specialisation. We may allow, if we will, that the men differ, so to speak, in *size*, one being twice as competent as another in *every* field, but the comparative capacities of all of them are the same. In these circumstances, if the process of specialisation never involves any cost, or if all sorts of specialisation involve equal costs, the maximum principle is satisfied when, and only when, the stock of people of working age is distributed among the several specialisms in such wise as to make the rates of pay for men of equal "size" equal, and for men of different sizes proportionate to size. If the processes of different sorts of specialisation involve different costs, the principle will be satisfied when, and only when, the rates of pay of men in different specialisms differ in correspondence—for the precise significance of this term the reader is referred forward to Chapter XXXV. § 5—with the differences in their several costs. For, failing this arrangement, the maximum principle is always pulling ultimate factors to move out of worse paid into better paid sub-forms. Needless to say, in actual life this pulling process does not completely attain its end. To imagine that it does would be, in Whewell's words, "as if the physical geographer should construct his scheme in recognition of gravitation alone, dis-

regarding the power of cohesion in preserving the original structure of the earth's surface, and should thus reach the conclusion that all the mountains must at once run down into the valleys and the face of nature become a plain".[1] It must be remembered, however, that we are here concerned with the economics of a stationary state, in which the frictions of actual life are thought away. For us the force of gravitation *is* supreme, save only when substantial and permanent obstacles obstruct its action. Apart, therefore, from such obstacles, so long as there are any stocks of both of two sub-factors, their maintenance prices are related in a definite and simple manner, however large or small the respective stocks may be. If conversion costs are not nil, the maintenance prices differ by a precisely determined amount. If conversion costs are nil, they are equal. In this case interdependence of maintenance prices is so close that it becomes identity. A minute change in the price of one sub-factor, the price of the other remaining constant, would mean an enormous upward (or downward) jump in its quantity, and a corresponding enormous downward (or upward) jump in the quantity of the other. Sub-factors related in this way are best regarded, not as two, but as one, subject to a single compound demand.

§ 4. *Prima facie* the preceding argument implies that there can never, for any particular sort of labour, be a separate maintenance function; but that, so long as there are any stocks of any two sorts, the prices of both must be either equal or divergent in a fixed degree. Even though this implication is relevant, not to actual life, but only to stationary states, it has a highly paradoxical appearance. Life, we are bound to think, cannot be as rigid as that. The solution of the difficulty is that a number of sub-factors, every unit of each of which is constructed out of one and the same type of ultimate factor, is not a proper model of the world. In truth many sub-factors contain some units manufactured out of one ultimate factor and some manufactured out of others. The men—and the same thing is, of course, true of pieces of land—that constitute the raw material of sub-factors, possess, not identical, but widely different comparative

[1] Quoted by F. A. Walker, *The Wages Question*, p. 175.

capacities, one being relatively better adapted for one kind of specialism and another for another. This fact destroys for the actual world the paradoxical implication suggested by our imaginary system. It introduces, however, several difficulties into our analysis. These are best attacked by stages. I shall first, therefore, in the main part of what follows, namely §§ 5-7, assume that no cost is entailed for anyone in the acquisition of any specialism, but that everyone's industrial qualities are borne with them full blown; and shall subsequently, in § 8, indicate the consequences that follow when this assumption is removed.

§ 5. Even when nil conversion costs are postulated the situation is very complex and must be treated with the help of simplifying assumptions. At the start, therefore, let us suppose that there are only two specialisms, say cake-stirring and cake-beating; and that there are a number of potential workers with various schemes of comparative capacity for the two. In these circumstances, when the prices offered for both of the two types of work are given, we may imagine a neutral man—it is immaterial whether such a man exists in fact or not—whose capacities are so adjusted that, in the given environment, with the given prices, he could earn an equal rate of pay at either kind of work. What precise sort of man occupies this neutral position depends, of course, on the state of prices. Every other man is turned to that specialism in respect of which his comparative capacity is greater than that of the neutral man. Thus, not merely how many men are devoted to either specialism, but also what individual men are devoted to each of them is determined.[1]

[1] In real life the adjustment we have been describing is, of course, often impeded by friction, so that particular men in their actual specialism are not getting as high pay as they could do in others. Thus, suppose that all captains in the Royal Navy are exactly worth their pay; that some of these captains would, as business men, be worth more than captain's pay; and that some of the men who fail to become captains would, as captains, be worth as much as any of the others, but, as business men, are worth much less than some of the others. The adjustment described in the text would require that no man should be retained as a captain who is worth more as a business man, and no man should be retained as a business man who is worth more as a captain. The first of these conditions *tends* to be satisfied by direct process, captains, who could do better outside, leaving the Navy—as Sir Otto Niemeyer left the Civil Service. But there is no direct process through

Men with no capacity at all for anything except one specialism—as the soldier class among certain species of ants, tied inescapably to soldiering by their physical constitution—must, of course, whatever the state of prices, adhere to that specialism, and can in no circumstances stand at, or, still less, move to the other side of, the position occupied by the neutral man.

§ 6. Next let us suppose that there are only two types of worker, all the members of each type possessing exactly similar capacity schemes; but a large number of specialisms. To make the discussion vivid let our two types of worker be men and women, the differences among individual members of both sexes being ignored. Let us suppose further that their respective schemes of comparative capacity are such that in any given specialism the absolute capacity of a man bears the same ratio to that of a woman, each being conceived as employed at the margin, whatever the state of demand and the output of the specialism may be. In these conditions we can draw up a scale that relates the absolute capacity of men and women over the whole field of occupations. Thus in specialism A a man produces ten times as much as a woman, and, at the opposite extreme, in specialism Z, a woman produces ten times as much as a man. At first sight it might seem that all the men will be assembled in the specialisms where their absolute capacity is greatest, and similarly all the women. But this is not so. The system of price offers per unit of efficiency in different specialisms and the schemes of capacity of men and women—*i.e.* the numbers of units of efficiency in respect of the several specialisms which they embody—being given, there will be some actual or hypothetical neutral specialism, in which both

which the second condition can be satisfied. There is only the indirect process of a reduction in captain's pay, brought about by pressure for employment by rejected men, leading, on the one hand, to the departure from captaincy of the existing captains who, now that captain's pay has been reduced, are worth more as business men, and, on the other hand, to the entrance into captaincy of all men who are now worth more as captains than as business men. But, of course, the rate of pay of naval captains is fixed in large part by tradition and convention, and in the actual world, as distinguished from an imaginary stationary state, is very unlikely ever to stand at the precise level which this adjustment requires.

men and women work. In this specialism their capacities will stand to one another in the ratio in which their wages stand everywhere. In all specialisms in which the relative capacities of men, as compared with women, are larger than in the neutral specialism only men will be assembled: in the other specialisms only women. In this way the quantities of labour power assembled in the two sets of specialisms are determined. If in the neutral specialism it happens that the absolute capacities of men and women are equal, the two sexes must everywhere receive equal wages; if in that specialism the absolute capacity of women is the larger, women's wages will be higher; and conversely. If the demand at current rates for the services at which women are absolutely more capable than men were greater than could be met by women alone, the neutral specialism would be an occupation in which the absolute capacity of women is larger than that of men. In actual fact the reverse of this condition holds. More of the services at which men are absolutely more capable than women are wanted than can be supplied by men alone. The neutral specialism, therefore, is one in which men are more capable absolutely; and men's wages are higher than women's wages.

§ 7. The third stage of our analysis consists in combining the considerations studied in the preceding two. There are now both a large number of specialisms and a large number of schemes of comparative capacity. The concepts of a neutral man and of a neutral occupation are both relevant. But, the system of price offers being given, there is not now only one neutral man or one neutral specialism. Rather, there is a neutral man as between every two kinds of specialism and a neutral specialism as between every two kinds of capacity. In respect of any given scheme of prices the stocks of men and women of each several initial quality will tend to be distributed among different specialisms in a specific manner, *i.e.* in such wise that nobody is receiving in the specialism to which he is attached a lower rate of pay than he would receive if transferred to any other.

§ 8. So far it has been assumed that any of the several specialisms that are open can be adopted without cost. If in

fact costs are involved, and if these are different for different specialisms, the conclusions of § 6 have to be qualified in the manner described at the end of § 3. This presents no difficulty. But there is a more serious complication. Up to this point we have tacitly assumed that, if training for any specialism costs anything at all, it costs a definite sum; that there are not available various degrees of training costing different sums. When this assumption is removed, the concept of a man's absolute capacity for any specialism, and, therefore, that of his comparative capacity for different specialisms, ceases to be clear-cut. The general environing conditions and his place at the margin being given, his capacity at any specialism cannot now be represented by a single quantity, but only by a series of quantities corresponding to different amounts spent on training. Of two men engaged, say, in the study of economics, one may prove the more capable if both are trained lightly, but the other if both are trained elaborately; just as, of two acres of land, one may be more fertile if both are given a small dressing of manure, but the other if both are given a large dressing. This consideration reveals, besides the sets of unknowns that we have already brought together, a further set depending on each particular man's individual suitability, not merely for different sorts, but for *different quantities of different sorts* of training. The system whose interrelated parts we have been studying is thus more complex than we have so far supposed. It is, however, not difficult to see that, along with the new unknowns, there are introduced an equal number of new equations; so that, in respect of any given set of wage offers, the maintenance situation is still determinate.

§ 9. The outcome of this discussion for the present purpose is plain. In the simplified model depicted in § 3 it appeared that the quantity of the supposedly single ultimate factor of production being given, no stock of any sub-factor could be maintained unless the rate of pay offered to it, allowance being made for costs of conversion of the ultimate factor into the several sub-factors, was exactly equal to the rate offered to any other; in other words that, the rates of pay of all the other sub-factors being given, the maintenance

function of any one of them was perfectly elastic. We have now seen that in actual conditions, where there are a number of different ultimate factors—*e.g.* men and women with different initial qualities—this is not so. If, as between two stationary states, the prices offered for all the other sub-factors being given, the price offered per unit of a particular sub-factor is reduced, this means, not that the stock of the sub-factor becomes nil, but merely that it becomes smaller than it was. That is to say, the prices ruling for all the other sub-factors being given, there is a maintenance curve for one of them sloping upwards with a finite positive elasticity. But this maintenance curve is not independent of the prices ruling for the other sub-factors. On the contrary, if these change, it of necessity changes also. In other words, the maintenance prices of the several kinds of labour constitute a mutually interconnected system.

CHAPTER XXX

THE INTERRELATIONS OF THE MAINTENANCE CONDITIONS OF LABOUR IN GENERAL AND CAPITAL (AND LAND) IN GENERAL

§ 1. BETWEEN the stocks at work, as defined in Chapter XXVIII., of labour in general on the one hand and capital (and land) on the other, there is a linkage, on the maintenance side, in respect of the number of hours worked per day. It is not, indeed, necessary that the working day of men and machines should be of equal length. It is possible for equipment to be worked continuously for 24 hours per day in association with two shifts of 12 hours or with three shifts of 8 hours for men; just as it is possible for two short shifts of boy labour to be worked along with one long shift of adult labour. But, though the lengths of working day for different factors need not be equal, they must obviously, on pain of chaos, be adjusted to one another in a precise manner. Thus, with a one-shift or a two-shift system, the length of hours for which men are ready to work determines the hours worked by machinery, and, if conditions connected with machinery make a 24-hours' shift for it imperative, this fact necessitates such length of working day for men as will allow of a conveniently adjusted system of shifts. In general, the hours of labour worked by machinery, as well as the hours worked by labour, are liable to be less, after adjustment has been made, when rates of pay to labour are high than when they are low: but the hours of neither factor are likely to be affected significantly by the rates of pay ruling for the services of machinery. Large results would obviously follow from the substitution of three 8-hour shifts for two—a change that

increases, in effect, the stock of capital equipment by 50 per cent. But issues connected with the length of the working day will not be further studied in this book. Attention will be confined to two forms of linkage that are more fundamental.

§ 2. The number of hours worked per day being given, there is a linkage between labour in general and capital (and land) in general, that is transitive, so to speak, from the latter to the former. The number of workers, whether hand workers or brain workers, that a given real price will maintain at work is not simply a function of that price, but depends also on the real income the workers receive as owners of capital and land; which in turn depends partly on how large the total income from these things is and partly on how the ownership of them is distributed. For, in so far as the economic situation reacts at all upon either the capacity per head or the number of any category of people, it must clearly do this through the amount of their total real incomes rather than through such part of these as they earn by work. In present conditions the main body of manual wage-earners own very little property. But the higher grades of brain workers own substantial quantities; and for them at least the point is important.

§ 3. There is also a linkage that is transitive from labour to capital. The stock of capital equipment which a given price will maintain is vitally dependent on the size of the stock of labour. For, the more labour there is, the larger is the marginal product of any given stock of capital. The representative man's rate of discounting future satisfactions being given, this implies that, other things being equal, in a stationary state associated with a large quantity of labour the capital stock must be bigger than in one associated with a small quantity. This linkage, to which attention has already been called in Chapter X. in respect of a Crusoe economy, is obviously a strong one and very important for practice.

CHAPTER XXXI

THE MAINTENANCE FUNCTION OF LABOUR IN GENERAL[1]

§ 1. In Chapter XXIX. we were concerned with the distribution of people, whose numbers and inborn qualities are given, into various types of specialised labour. If the stocks of people with each sort of inborn quality were in fact fixed, as in that chapter we provisionally assumed them to be, there would be nothing more to say. But, of course, these stocks are not fixed. The rates of pay offered to the various sub-factors affect their several quantities, not merely by determining the distribution of the ultimate factors among them, but also by determining how much of these there is to distribute. Now, therefore, ignoring obstacles to adjustment and postulating that distribution has been made in accordance with the maximum principle, we have to confront the stock of each ultimate factor with the (common) rate of pay offered for it in no matter what sub-form. For a full analysis we should need to bring into account all the various groups of people whose inborn qualities are different. This is plainly impossible. Men and women are not segregated into "pure lines". There is no clear-cut relation between the rates of pay to people of musical attainments in one generation and the number with musical capacities who will be born in the next. Moreover, there are grave complications about joint production. The numbers of men and women that are maintained do not depend severally upon the rates of pay offered to each sex, but rather upon the combined wages of men and women. Among bees, indeed, there is a mysterious mechanism, by which the queen can determine which eggs shall be

[1] Cf. Appendix VIII.

fertilised and produce female workers and which unfertilised and produce males. But among human beings there is no corresponding mechanism, and, except in nations which indulge in female infanticide, there is no connection between the relative numbers of men and women and their relative rates of pay. In view of these and other difficulties, we must perforce make a violent abstraction and, in the manner of the classical economists, attempt a bird's-eye view of the relation between rates of wages and stocks of labour as a whole. We have, that is to say, to envisage the maintenance function of labour in general.

§ 2. This function is made complex by the fact that the stock of labour is, so to speak, three-dimensional. Its magnitude depends partly on the length of hours per day for which the representative worker works, partly on the capacity of —the amount of work power stored in—the representative worker, and partly on the number of workers. We have, therefore, to bring into account all these three elements. They are not, of course, independent of one another. The length of the working day and the capacity of the representative worker affect the rate of earnings per man; and this rate of earnings may affect the number of workers. In like manner the number of workers affects the rate of pay per hour's work when any given length of working day is being followed; and this rate of pay reacts both on capacity and on the length of the working day. This complication cannot, however, be further discussed. I shall content myself with separate studies of the three elements distinguished above.

§ 3. Let us begin with the relation between real wage rates and the number of hours per day, or, more generally, so to allow for variations in intensity, with the relation between real wage rates and the "quantity" of work that a representative worker of given capacity is willing to perform. Higher rates of pay evidently *may* be associated with larger quantities of work: but they may also be associated with smaller quantities. It is not difficult to see in a general way that, even when our representative worker's desire for commodity—there is, it will be remembered, only one sort of commodity—and aversion from work are independent, this

is possible. For suppose that his desire for commodity was already, before his rate of pay improved, practically satisfied. If he were to go on working as much as before, he would only obtain his former satisfaction from consumption, while, by working less, he could save himself a substantial direct disutility. In these circumstances it will be to his interest to work less. It may well be, however, that his aversion attitude to work is not independent of his holding of product, but that an enlarged holding, by giving him more means to enjoy leisure, increases his dislike for (any given quantity of) work. When this is so, the likelihood of higher rates of pay being associated with smaller quantities of work is *pro tanto* stronger. Whether this is so or not, the likelihood is clearly strong if the elasticity, over the relevant range, of his marginal desire for commodity is very small, and weak if it is very large. Now, in respect of very short periods, a man's standard of life is apt to be more or less fixed. This means that his marginal desire for commodity is very inelastic. Hence it is not surprising that in coal-mining the immediate consequence of a rise in piece-wages has often been a reduction in the output of work. But with time and experience people adapt their standards to their environment. From the standpoint of a normal period, therefore, their marginal desire for commodity-income is apt to be considerably elastic, so that the association of higher prices with less work is not so probable. The historical movement towards a shortening of the hours of labour, as general prosperity has increased, suggests that this relation has played an important part in actual life. It must be borne in mind, however, that shorter hours may be accompanied by increased intensity of work, and so do not necessarily imply a smaller quantity of work, in the sense defined at the beginning of this section.

§ 4. I turn to the second element distinguished in § 2, the relation between rates of pay and our representative worker's capacity for work—the number of capacity units, so to speak, that are embodied in him. There can be no doubt that, in the lower income ranges, higher income, by enabling a man to be better fed, housed, clothed, and, may be, educated—I am abandoning for the moment the rigid assumption that one

commodity only enters into income—thereby enables him, in a given time and with a given subjective effort, to perform more work of a given quality: and it is also almost certainly true that the enjoyment of a reasonable quantity of leisure has a like consequence. Even apart from the point about leisure, since he is certain not to do so much less work that his income is reduced below what it was before, higher pay may be expected to enhance his capacity.[1]

§ 5. Before we pass to my third element, the number of workers, mention may be made of an *indirect* reaction on capacity that may in some circumstances emerge. The average capacity of a given number of people of working age —conventionally set as between 15 and 64—depends in part on how these people are distributed among different ages. In most manual occupations, for example, a worker in the early sixties embodies less working power than one in the early thirties. If, therefore, different rates of pay brought about different age-distributions among people of working age, the amount of labour power would not vary simply in proportion to the number of these people. It will be argued in a moment, however, that, as between stationary states, different rates of pay have no tendency to affect the proportions of the population who are respectively within and without of working age. If this is correct, the same thing is presumably true of age-distribution within working age. I turn, therefore, to the relation between rates of pay and the proportion of the population that is of working age.

§ 6. Everybody knows that, when in actual life changes take place in death-rates or in birth-rates, the age-distribution of the population is liable, for a number of years, to be greatly modified. In Great Britain, for example, the proportion of the population aged 15 to 64 was 66 per cent in 1911, 68 per cent in 1921 and 70 per cent in 1931: the corresponding percentages of persons under 14 being 28·9 per cent, 26 per cent and 22·6 per cent, and those of persons over

[1] In the higher income ranges it may happen that enlarged income, by breeding over-indulgence, *lessens* capacity. But what has been said in the text may fairly be taken to represent the general case, at all events from the long-period standpoint proper to the study of stationary states.

64, 5·2 per cent, 6 per cent and 7·4 per cent. So far, therefore, as changes in the relevant death- and birth-rates are a consequence of changes in rates of pay, substantial changes in age-distribution may also be a consequence of them. We are concerned here, however, with the relation between rates of pay and age-distribution, not during a process of movement, but as between stationary states. Now, as we saw in Chapter IV. § 2, a *conditio sine qua non* of a stationary state is that the number of people aged n years who die every year is always equal to the excess of the number of people (n) years old over the number ($n+1$) years old. For, if this condition is not satisfied, the age-distribution of the population is undergoing change, and so the state is not stationary. As between two stationary states, therefore, whether they have the same or different birth-rates, age-distribution can only be different *if the proportions between the death-rates of different ages are different*, if, for example, in the one state the death-rate of people under 14 divided by the death-rate of people over 14 is higher or lower than in the other. It is well known that in this country in recent times the rates of mortality of infants have been reduced in a much larger proportion than other rates. But this is, in the main, a consequence of advance in sanitary and medical science, and has probably little to do with rates of pay. On the whole, there is no reason to suppose that, as between stationary states, different rates of pay will be associated with appreciably different schemes of relative death-rates. For the purposes of our problem, therefore, the number of persons of working age may be conceived as constituting a fixed proportion of the population: and attention may be concentrated on the relation between rates of pay and total population.

§ 7. The fundamental fact to seize upon is that in every stationary state birth-rates and death-rates must be equal. Recognising this, we can best attack our problem by imagining an initial stationary state in which population is such and such and rate of pay so and so, and then supposing that something—an improvement in productivity or what we will—causes the demand for labour to increase. Stationariness is, of course, at once destroyed, and a process towards a second

different stationary state is set up. Let us provisionally, as a first step, suppose that the death-rate is not made different from what it was originally, either at any point in the course of this process or at the conclusion of it. In the new stationary state, then, the birth-rate must be the same as in the original one; in more homely terms, the average size of people's families must be the same. The relation between the numbers of the population in the new and the old stationary state depends, therefore, on what has happened *during the process of transition* between the two states. When the birth-rate comes back to what it was originally, the number existing when it comes back is gripped into stationariness. Will this number be larger than, the same as, or less than the original number? Three cases may be distinguished. First, if higher pay leads, throughout the period of transition, to a higher birth-rate—larger families—the population must grow continually over this period, and the new stationary state cannot be attained until the rate of pay has fallen again to what it was originally. This is the situation contemplated by the so-called iron law of wages; the maintenance curve for population numbers is a horizontal straight line. Secondly, if after a time better pay reacts on the standard of life, so as to raise it, but to raise it less than in proportion to the rate of pay, the new stationary state will again contain a larger population than before; but this time there will also rule in it a higher rate of pay. That is to say, the maintenance curve of population is a line sloping upwards towards the right. Thirdly, higher pay may so react on standards as to lift them more than in proportion to pay. If this happens, when the rate of pay rises, the size of families will presently decline below what it was in the original stationary state. This means that in the new stationary state there will be a smaller population and a higher rate of pay than in the original one. The maintenance curve for population will bend over backwards towards the vertical axis, *i.e.* it will be backward-rising. Such historical information as is available suggests that the second of these possibilities—larger population *plus* a higher rate of pay in the new stationary state—is the one most likely to be realised

in fact. In any event it is impossible that in the new stationary state there should be a larger population *plus* a lower rate of pay, *i.e.* that the maintenance curve should slope downwards towards the right, *i.e.* be forward-falling.

§ 8. It only remains to remove our provisional assumption that differences in the rate of pay are associated with no differences in the death-rate. It is, of course, not to be supposed that improved pay *increases* the death-rate, but it may well diminish it. If it does, everything that has been said still holds good. In the new stationary state the birth-rate, instead of being the same as it was in the original one, is lower than it was. But the possible and probable relations between rates of pay and numbers of the population are not affected.

§ 9. To derive from the maintenance function for population that for labour power at work we have to combine this analysis with what was said in §§ 3-4 about the reactions of higher rates of pay upon hours of work and individual capacity. If the iron law rules, so that in the new stationary state the rate of pay is the same as in the old, there is, of course, no place for these reactions. But, if the rate of pay in the new stationary state is higher—we have seen that it cannot be lower—than in the old, they play a part. There is, as we have seen, little doubt that the average worker's capacity will be expanded. On the other hand, the argument of § 3 suggests that the length of the working day will *probably* be reduced. Should the former effect predominate, the maintenance curve for labour power at work cannot be backward-rising if the maintenance curve for population is forward-rising, and may be forward-rising even though that curve is backward-rising. Should the tendency to shortened hours predominate over the tendency to enhanced capacity, converse propositions hold. There are no means of deciding which of these two tendencies is likely in fact to predominate. Since, then, we have already concluded that the maintenance curve for population is unlikely to be backward-rising, we must also conclude that, on the evidence available, the maintenance curve for labour power at work is unlikely to be of that form. We may expect it, the stock of capital being given, to be forward-rising in a moderate degree.

CHAPTER XXXII

THE MAINTENANCE FUNCTION OF CAPITAL IN GENERAL

§ 1. THE maintenance function of capital in respect of stationary states is best studied on the lines followed in §§ 6-8 of the preceding chapter. This stock of capital experiences a death-rate in the equipment that annually wears out and a birth-rate in the gross new equipment annually created. The stock is stationary when death-rate and birth-rate are equal. These two rates are equal when the total stock is such that its marginal product, expressed, in the manner of Chapter X. § 6, as a rate of interest, is equal to the rate at which *the representative man possessed of representative income* —a concept not, indeed, quite precise but adequate for our purpose—discounts future satisfactions. This conclusion is established by the analysis of Chapter X.

§ 2. The effect on the capacity for work of the owners of capital of variations in their real income, to which reference was made in Chapter XXX. § 2, is probably of secondary importance, and will be ignored. On this basis it is easy to see that, if the rate at which people discount future satisfactions is independent of their capital holdings and annual incomes, the maintenance price is one and the same for all sizes of stock. There is thus an iron law of capital analogous to the once popular iron law of wages; in other words, the maintenance curve for capital is a horizontal straight line. If, on the other hand, large capital holdings and annual income have the effect of reducing the rate at which people discount future satisfactions, the maintenance price of capital must be smaller in stationary states with large stocks than in stationary states with small stocks; that is to say, the

maintenance curve for capital slopes downwards towards the right, *i.e.* is forward-falling. If this is not obvious, it can be demonstrated by means of an argument analogous to that used in § 7 of the preceding chapter. Suppose that we start from one stationary state with a fixed capital stock and such and such a rate of interest. The real demand function for capital is lifted. The rate of interest thereupon rises, and therefore, as is implied in what was said in § 1, the annual net creation of capital rises from nothing to some positive quantity. The stock of capital thus grows and, as it grows, since the elasticity of demand is negative, its marginal private product falls. This means that the rate of interest falls. But, when this has become equal again to the original rate of discounting future satisfactions, since that rate has meanwhile fallen, equilibrium is not attained. The stock of capital must be further increased till the rate of interest becomes equal to the new lower rate of discounting future satisfactions that has been established: which was the proposition to be proved. Now nobody can seriously suppose that capital holdings or large incomes have the effect of *increasing* the rate at which people discount the future. Any third form of maintenance curve may, therefore, be ruled out of account. The actual curve must be either horizontal or forward-falling. Which of these two forms it has depends on whether in fact large capital holdings and large annual incomes promote a low rate of discounting future satisfactions.

§ 3. If we were concerned merely with enlargements in aggregate capital holdings and aggregate annual incomes associated with proportionate enlargements in the number of persons enjoying them, there would obviously be no question of any reaction on people's rate of discounting future satisfactions. Here, however, we are considering the maintenance function of capital, subject to the condition *other things being equal*. Enlargements in aggregate capital holdings and annual incomes must, therefore, be conceived as carrying with them equiproportional enlargements per head.[1] In these circumstances, it is by no means self-evident that

[1] What is relevant here is, of course, aggregate income per head, not merely income per head derived from capital. Since, other things being

enlargement leaves the representative man's rate of discounting future satisfactions unaffected. On the contrary, the point needs careful discussion.

§ 4. It is a familiar fact that in actual life people with large incomes tend to save annually, not merely larger absolute amounts, but larger proportions of their incomes than people with small incomes. From this it is a natural inference that the rate of interest required to induce a man of given mentality to keep a large capital exactly intact is likely to be lower than that required to induce a similar man to keep a small capital exactly intact: and this again suggests that the rate at which the former discounts future satisfactions is lower than that at which the latter discounts them. This inference would, however, only be warranted if the proportionate effect on the marginal utility—desiredness—of income associated with small changes in its amount were no greater for people with small than for people with large incomes; whereas there can be little doubt that in fact the opposite of this is true. If that be so, it can be shown that people with large incomes will save, for a given rate of interest, a larger proportion of them than people with small incomes, even though their rates of discounting future satisfactions are identical.[1] It follows that no conclusion about the relations between the rates of discounting future satisfactions of people with large and small incomes can be drawn from the fact that the former class make proportionally larger savings.

§ 5. The question, therefore, whether largeness of income is in fact likely to promote a low rate of discounting future satisfactions can only be answered by means of a direct judgment. I suggest that, when incomes are very low, people's minds are of necessity so concentrated on the urgencies of the present moment that the future looms for them very small. Their rate of discounting future satisfactions is likely to be high. With larger incomes there is oppor-

equal, an enlargement in the stock of capital per head necessarily entails an enlargement in the former quantity, it is immaterial that it *may* entail a contraction in the latter.

[1] This point, which is difficult to grasp from a verbal exposition, is worked out with symbols in Appendix IX.

tunity for a wider view and a more educated appreciation of the facts of life. But, after incomes have reached a certain moderate size, further enlargements are not likely to make much difference. I conclude that the maintenance curve for capital, the stock of labour power being given, is probably forward-falling in the earlier part of its course, *i.e.* in respect of small stocks of capital, but in respect of large stocks becomes approximately a horizontal line.

CHAPTER XXXIII

EQUILIBRIUM OF DEMAND AND MAINTENANCE IN A STATIONARY STATE[1]

§ 1. CHAPTERS XXIII.-XXVII. studied demand prices, Chapters XXVIII.-XXXII. maintenance prices. The demand prices for the several factors and sub-factors are not independent of one another. In general, each demand price for a given quantity of factor or sub-factor is what it is only provided that the quantities of the other factors and sub-factors at work are what they are. The same thing is true of the maintenance prices of the several factors and sub-factors. On both sides alike we have to do, not with a number of independent isolated elements, but with a complex system of interconnected elements. Equilibrium in a stationary state requires that these two systems should fit together.

§ 2. Let us first suppose that monopoly power is nowhere exercised. Our analysis of demand has shown that the prices —demand prices—in terms of product of the several factors and sub-factors are determined when the quantities are given: our analysis of maintenance that the quantities are determined when the prices—maintenance prices—are given. The quantities determine the prices, and at the same time the prices mutually determine the quantities. The system, therefore, is in equilibrium if, and only if, the several quantities are such that the demand price of each is equal to its maintenance price, *i.e.* to the rate of pay that will precisely maintain the existing quantity at work. If these conditions are satisfied, but not otherwise, there is a constant output of cake and a constant rate of pay in cake for each factor of production.

[1] Cf. Appendix X.

§ 3. If monopoly power is exercised by one person or set of persons among those controlling one factor or sub-factor, the conditions of equilibrium are different. The rate of pay which the monopolistic factor receives is still equal to the demand price of the quantity of it at work. But it is not now equal to the maintenance price of that quantity. The quantity is kept down below what it would be if there were no monopolistic action, in such wise as to make the maintenance price of it less than the demand price. The extent to which it is thus kept down depends, of course, in part on how large a proportion of the factor in question is subjected to monopolistic action. If the whole of it is so subjected, the quantity maintained at work will be such that every representative owner of bits of the factor sets to work that quantity in respect of which his aversion from the marginal unit of work is equal, not to his desire for the product of that unit, but to his desire for the addition made by it to the total receipts of the factor. This, in general, entails a smaller amount of work for the factor. It is easy to see in a general way, and can readily be proved algebraically, that, the smaller the proportion of the factor that is subject to monopolistic action, the less far the quantity of it at work will be contracted below what it would have been had there been no monopoly. But, if any part is subject to monopolistic action, there is bound to be some contraction. Furthermore, it must be held in mind, monopolistic action in respect of one factor, besides affecting directly the quantity of that factor at work, also affects indirectly the quantities of the other factors. For it makes the demand prices of given quantities of them other than, in general less than, they would have been without it; and this leads to a new adjustment between demand and maintenance prices, probably, though, as will appear presently, not necessarily, entailing smaller quantities of most other factors. There is some presumption, however, that the damage to total output due to these secondary reactions of monopoly will be small relatively to those due to its primary reactions.

§ 4. In the preceding section it was tacitly assumed that units of one factor or sub-factor cannot transform themselves into units of another. As was shown, however, in Chapter

XXIX., as between different sub-factors transformation can take place. It is, therefore, necessary to distinguish between types of monopolistic action according to the way in which they affect transformation. A monopolistic body, which restricts the stock of the sub-factor it controls by directly limiting the quantities of the ultimate factors out of which it is built that come into it—*e.g.* through regulations about apprentices, the imposition of large premiums on entrance, the creation of an education test not relevant to competence in the craft—obviously leaves free an equal quantity of ultimate factors to turn themselves into other kinds of sub-factors. Thus, if doctors limit their numbers, this directly, apart altogether from indirect effects on the size of the population, causes the number of lawyers to be larger than it would otherwise have been; and so on. The quantity of other sub-factors at work is, other things being equal, *pro tanto* augmented. Where a monopolistic body operates by forcing up the rates of payment for which the sub-factor it controls will work, *e.g.* by minimum wage regulations, the consequences depend on the method by which units of the sub-factor in question are engaged. If these methods are such that there is no possibility of part-time employment, but every unit seeking work is accepted or rejected once for all, the effect is precisely the same as that of restrictions upon entrance. But, with methods of engagement such that every unit of the sub-factor has much the same expectation of employment as every other, a gap will appear between the quantity of the sub-factor that is employed and the quantity that is created. The rate of pay that is enforced will determine the quantity that is employed without respect to the quantity that is created: while such quantity will be created that this quantity divided by the quantity employed is equal, roughly, to the enforced rate of pay divided by the rate of pay that rules for units of the factor, out of which the sub-factor is made, that are incorporated in other sub-factors. The final result then as regards the quantity of the sub-factor actually in action is the same as would be produced by directly limiting the amount of it employed in the degree in which it is in fact indirectly limited. But the addition indirectly caused to the

quantity of the other sub-factors in existence is cut below what it would have been to the extent of the pool of unemployed sub-factor that has been created. It should be noted that, if the demand for the restricting sub-factor has an elasticity less than unity, more of the fundamental factor will be incorporated in it than would have been incorporated with nil restriction. Not only is some of the sub-factor that would have been there in any event wasted, but further units of the fundamental factor, which would have belonged to and been employed in other sub-factors, are dragged out of them, to be attached in idleness to the restricting sub-factor. This effect is a familiar one in the field of labour. It also sometimes follows, in respect of capital equipment, from kartel agreements in industries of highly inelastic demand. In those industries extra firms come to be established and equipment is created surplus to needs; while there is less equipment in other industries than there would have been had no kartels been formed. The actual situation thus always differs from that described in the preceding section, save only in the special case where the cut caused by monopolistic action in the quantity of the monopolised sub-factor at work is exactly offset by an equivalent enhancement in the quantity of that sub-factor out of work.

§ 5. If monopoly power is exercised by more than one person or group of persons, controlling different parts of the same factor or sub-factor, the quantities of the factor that will be at work are mathematically, and therefore, on the data, economically indeterminate: but, if monopoly power is exercised by more than one person or group, controlling different sub-factors, the situation need not be indeterminate, provided that the number of sub-factors is large. In any event every factor and sub-factor will be paid a price equal to its marginal product. These conclusions follow from the general argument of Chapters XVII. and XIX.

CHAPTER XXXIV

THE REMUNERATIONS OF THE SEVERAL FACTORS AND SUB-FACTORS OF PRODUCTION IN A ONE-COMMODITY COMMUNITY IN DIFFERENT STATIONARY STATES ON THE ASSUMPTION THAT THERE ARE NO ECONOMIES OR DIS-ECONOMIES OF LARGE SCALE [1]

§ 1. THE purpose of this chapter is to compare the real remunerations, absolute and relative, which the several factors —I now use this term in a wide sense to include sub-factors— of production receive as between two stationary states in a one-commodity community that differ from one another in certain assigned particulars. These states, it is important to realise—the word stationary, indeed, implies it—are conceived as standing in equilibrium. We are not concerned with the process of transition that takes place *after* one equilibrium situation has been disturbed by an alteration in the elements governing demand or maintenance and *before* the new equilibrium situation appropriate to the new conditions has been reached. If A's maintenance function is changed, we have, as a first result, some change in A's quantity; hence in B's rate of remuneration; hence in B's quantity. This in turn reacts on A's quantity; and a series of adjustments occur, in such wise that the ultimate difference in A's quantity is not the same as the difference that emerged immediately after the shift in A's maintenance function; and similarly with B's quantity. In like manner, for any other initial shift that is made a series of adjustments take place before the new stationary state is attained. It is the new state itself in relation to the old, not the intervening process,

[1] Cf. Appendix XI.

that we are called upon to study. Thus any such suggestion as that an increase in the stock of capital must benefit labour because it is only with the help of labour that additions to capital equipment can be made is here beside the point.

§ 2. In the discussion that follows the assumptions of the preceding chapters, that movement from place to place is completely unhampered and costs of transport nil, are, of course, still maintained. The productivity function is assumed, in accordance with what was said in Chapter XXVII. § 10, to be homogeneous in the first degree—*i.e.*, such that there are no economies or diseconomies of large scale—and also to be related to diminishing returns to individual factors of production in the manner described in Chapter XXIV. § 6. On the maintenance side a distinction is drawn between cases in which the quantity of each factor that is maintained respectively does and does not depend on the real price offered to itself alone, regardless of what is offered to the others. So much being understood, I shall first postulate that the productivity function is given, and shall examine, on that basis, the consequences of a change in the maintenance conditions affecting one of the factors and leading to an increase in its quantity. Thereafter, I shall postulate that all the maintenance conditions are given and shall suppose that the productivity function changes.

§ 3. Before these issues are attacked it is desirable to bring together the definitions of certain relevant elasticities, to some of which reference has already been made, while others here come upon the scene for the first time. We may conveniently begin with those on the demand side. The elasticity of total marginal productivity of x units of any factor of production X—to be called E_x—in respect of the commodity that it is helping to produce is the (small) proportionate change in the quantity of the factor divided by the associated proportionate change in its marginal physical productivity, when its quantity has been altered by a shift in its maintenance conditions *and all the required adjustments in the quantities of the other factors have been made.* The elasticity of partial marginal productivity of x units of X in respect of changes in x, namely ${}_x\epsilon_x$, is the (small) proportionate change in x

divided by the proportionate change in X's marginal productivity that would have taken place had all the factors other than X been held constant. In like manner the elasticity of partial marginal productivity of y units of any other factor Y in respect of changes in X, namely ${}_x\epsilon_y$, is the (small) proportionate change in x divided by the proportionate change in Y's marginal productivity that would have taken place had all the factors other than X been held constant. The rule of diminishing returns to individual factors of production, cited in the preceding section, tells us that elasticity of partial marginal productivity of every factor in respect of itself is negative. That is to say ${}_x\epsilon_x$, ${}_y\epsilon_y$, ${}_z\epsilon_z$ and so on are all negative. Throughout what follows I shall postulate that, so far as it affects our problem, this rule is universally obeyed.

§ 4. On the maintenance side, for x units of any factor of production X, there is an elasticity of maintenance in respect of changes in x, namely ${}_xe_x$, measured by the (small) proportionate change in x, divided, all other things remaining the same, by the associated proportionate change in the maintenance price of X; there is an elasticity of maintenance of X in respect of changes in y (the quantity of Y), namely ${}_ye_x$ measured by the (small) proportionate change in y divided, all other things remaining the same, by the associated proportionate change in the maintenance price of X. In like manner for the factor of production Y there is an elasticity of maintenance in respect of changes in y, namely ${}_ye_y$, and one in respect of changes in x, namely ${}_xe_y$. In general, for any factor R there is an elasticity of maintenance in respect of changes in the quantity of any (the same or another) factor Q, which may be represented by the expression ${}_qe_r$.

§ 5. There is no difficulty in understanding the significance of the elasticity of maintenance of a factor in respect of changes in the quantity of itself. It is formally equivalent to the elasticity of supply as ordinarily conceived. *A priori* it may be either positive or negative. If it is positive over a given range, in respect of that range the maintenance curve (drawn, of course, subject to the condition, other things being equal) will slope upwards throughout from left to right, *i.e.* will be forward-rising. In this case a rise through-

out its length of the related demand curve entails a higher price and a larger stock. If the elasticity is negative, either of two things, as was shown in Chapter XXXI. § 7, is possible. First, over the relevant range the maintenance curve may slope downwards from left to right: it may be forward-falling. A rise throughout its length of the related demand curve then entails a lower price and a larger stock. Secondly, over the relevant range the maintenance curve may bend backwards from right to left, rising as it moves towards the left: it may be backward-rising. In this case a rise throughout its length of the related demand curve entails a higher price and a smaller stock. It is impossible that there should emerge, as a consequence of the demand curve rising, a lower price and a smaller stock. In Chapters XXXI.-XXXII. it has been suggested that the maintenance curve of labour in general, when the stock of capital is given, is probably forward-rising and has a positive elasticity, while that of capital in general, when the stock of labour is given, is probably forward-falling with a negative elasticity. From the standpoint of a short period it may often happen that factors or sub-factors have maintenance functions of negative elasticity and are backward-rising, but maintenance functions of this kind are clearly much less likely to be found in the long period that alone is relevant to stationary states.

§ 6. The concept, elasticity of maintenance of one factor in respect of changes in the quantity of another factor, is less familiar. The elasticity of maintenance of Y in respect of changes in the quantity of X is defined, as was stated above, as the (small) proportionate change in the quantity of X divided by the associated proportionate change, other things being equal, in the maintenance price of the existing quantity of Y. If then this maintenance elasticity ${}_xe_y$ is positive, an increase in the quantity of X will entail a rise in the price required to maintain a given quantity of Y, and therefore, with a forward-rising or forward-falling maintenance curve for Y, a decrease in the quantity of Y that is maintained with a given demand curve. Thus, an increase in the number of coal-miners, all other things being the same, entails a decrease in the number of people trained to rival occupations.

If on the other hand ${}_xe_y$ is negative, an increase in the quantity of X will entail a fall in the price required to maintain a given quantity of Y, and therefore, with a forward-rising or forward-falling maintenance curve for Y, an increase in the quantity of Y. For example, an increase in the stock of labour in general entails, all other things being equal, an increase in the stock of capital. Clearly, so far as *a priori* considerations go, ${}_xe_y$ and all the other elasticities of like type may be either positive or negative. If the maintenance prices of all the factors are independent of one another, changes in the quantity of any one factor leave the maintenance price of every other factor unaltered. The elasticity ${}_xe_y$ and all the other e's for which the two suffixes differ are then infinite, and their reciprocals are nil. They have no relevance to any problem; and we are left with one maintenance elasticity only for each factor, ${}_xe_x$ for X, ${}_ye_y$ for Y, and so on. In what follows the concepts symbolised by ${}_xe_y$, ${}_ye_x$ and so on will not be employed in studying the consequences of a shift in the quantity of one sub-factor between which and some other sub-factor transformation can take place absolutely freely or even at a fixed cost per unit. As was argued in Chapter XXIX. § 3, such sub-factors are best regarded, not as two sub-factors, but as one only, subject to a single compounded demand.

§ 7. With these preliminaries, let us, in accordance with the programme of § 2, begin by considering the effect on the absolute remuneration of one factor of production X when, in consequence of a shift in the maintenance conditions affecting it and not affecting directly the quantity of any other factor, its quantity, when the appropriate new equilibrium is established, is enlarged beyond what it was originally. It is self-evident that the aggregate remuneration of this factor will be increased or diminished according as its elasticity of total marginal productivity is, on the one hand, either positive or, being negative, numerically greater than unity, or, on the other hand, is negative and numerically less than unity over the relevant range.[1] This proposition is

[1] Of course, if the elasticity is >1 at some points and <1 at other points over the relevant range, the total result can only be determined when the elasticities at all the points are known.

a very familiar one and calls for no discussion. It is, however, of interest to set out the conditions upon which the magnitude of this elasticity, and so the likelihood that it will be positive or, being negative, numerically greater than unity, depends. If the quantities of all the other factors are rigidly fixed, the elasticity of total marginal productivity of the factor X is obviously identical with the elasticity of partial marginal productivity in respect of changes in itself. It thus depends simply on the characteristics of the general productivity function. It cannot be positive, and its negative value may be numerically either greater or less than unity. There is nothing further to be said about it. If the quantities of the other factors are not rigidly fixed, the magnitude of the elasticity of total marginal productivity of factor X is not equal to the elasticity of partial marginal productivity in respect of changes in itself; but depends partly on this and partly on a number of other things. First, there are the series of elasticities of partial marginal productivity, ${}_{x}\epsilon_{x}$, ${}_{y}\epsilon_{x}$ and so on, which are given in the constitution of the general productivity function and which are thus ultimate elements. Secondly, there are the series of quantities dy/dx, dz/dx and so on, which measure the consequential changes induced in the quantities of the other factors of production by a change in the quantity of X. These elements in turn are partly dependent on the several elasticities of partial marginal productivity, and partly on the several elasticities of maintenance other than the elasticities of maintenance relating to X. It follows that they are only mediating elements, and that ultimately the value of E_x depends entirely on the values of the several elasticities of partial marginal productivity and the several elasticities of maintenance other than those relating to X. It is easy to see that, with n factors of production, there must be n^2 elasticities of partial marginal productivity, and $(n^2 - n)$ elasticities of maintenance that do not relate to X. If the maintenance functions of the several factors are all independent of one another, all the elasticities of maintenance are infinite and their reciprocals nil—which entails that they do not affect the value of E_x – except only the elasticity of maintenance of each factor in respect of changes in its own

quantity. These are, of course, n in number, of which one is related to X. Thus, with interdependent maintenance functions, the value of E_x depends on the values of n^2 elasticities of partial marginal productivity and of $(n^2 - n)$ elasticities of maintenance; with independent maintenance functions upon the values of n^2 elasticities of partial marginal productivity and $(n - 1)$ elasticities of maintenance. The precise nature of this relationship is examined in the case of three factors—with a larger number of factors no new principle is introduced—in Appendix XI. In this case the situation is too complex to allow of any simple generalisations. The formulae must speak for themselves. When, however, there are only two factors X and Y, straightforward translations can be made. With the maintenance conditions of X and Y independent, if ${}_ye_y$ is positive, E_x must be negative. It then has a larger negative value, and so is more likely to be numerically larger than unity, the larger numerically are ${}_x\epsilon_x$ and ${}_ye_y$, and the smaller numerically is ${}_y\epsilon_y$. If ${}_ye_y$ is negative on a forward-falling curve, the conditions of stable equilibrium are shown in Appendix XI. §§ 8-9 to require that E_x shall be positive. If ${}_ye_y$ is negative on a backward-rising curve, E_x must be negative, its negative value being larger the larger numerically are ${}_xe_x$ and ${}_y\epsilon_y$, and the smaller numerically is ${}_ye_y$. If interdependence of maintenance conditions rules instead of independence, this makes it more likely that E_x will be a positive or a large negative quantity when ${}_xe_y$ is negative, and has the contrary effect when ${}_xe_y$ is positive. This is mere common-sense; for, as we have seen, ${}_xe_y$ being negative means that an enhancement in the quantity of X directly entails an expansion in the quantity of Y, and this, of course, entails an enhancement in the total marginal productivity of X.

§ 8. Let us now turn to the effect of a shift in the maintenance conditions of one factor, leading to an increase in its quantity, upon the remunerations of other factors. Consider first the relatively simple case in which the quantities of all the factors other than X are rigidly fixed. Obviously the question whether the aggregate remuneration of any one factor—call it Y—is increased or not depends on whether its

total marginal productivity is increased or not. If there are only two factors, it is easy to show that the total marginal productivity, and so the remuneration, of Y must be increased. X being the quantity and p_x the initial rate of the pay of the variable factor, when the quantity of that factor increases by ΔX, total output increases (approximately) by $\Delta_x . p_x$, while the total remuneration of X increases by $\{\Delta X . p_x + X . \Delta p_x\}$. Since Δp_x is negative, the latter quantity, which, as we have seen, may be either positive or negative, is necessarily less than the former. It follows that the total remuneration of the factor Y must be increased. In the limiting case where X and Y are perfect substitutes, *i.e.* identical for all relevant purposes, since, *ex hypothesi*, there are no economies or diseconomies of large scale, the total remuneration of Y is not altered by an increase in X. If there are more than two factors, the fact that an increase in X entails an addition to total output larger than the addition made to X's remuneration suffices to show that—except in the limiting case—the aggregate remuneration of all the other factors together must be increased, but not that the aggregate remuneration of any particular one must be increased. For any particular one, say Y, total remuneration will be increased or diminished according as its elasticity of partial marginal productivity in respect of shifts in X, namely ${}_x\epsilon_y$ is positive or negative. The argument has been in effect a proof that, when there are only two factors, ${}_x\epsilon_y$ is necessarily positive. When there are more than two factors there is clearly no necessity for this. In the case of three factors it is impossible that both the elasticities ${}_x\epsilon_y$ and ${}_z\epsilon_y$ shall be negative; but, while both *may be* positive, there is nothing to prevent either the one or the other from being negative.[1] Whether either, and, if so, which, of them is negative is sheer matter of fact, depending on the form of the productivity function. The only general proposition that can be laid down is the somewhat futile one, based on the rule of diminishing returns for individual factors, that, if the

[1] For, as is shown in Appendix XI. § 6, if $F(x, y, z)$ is homogeneous in the first degree, $\left\{\frac{1}{{}_y\epsilon_y} + \frac{1}{{}_x\epsilon_y} + \frac{1}{{}_z\epsilon_y}\right\} = 0$ and, from the rule of diminishing returns to individual factors of production, $1/{}_y\epsilon_y$ is negative.

factor X is "identical" with any other factor Y, ${}_x\epsilon_y$ must be negative.[1]

§ 9. Let us now withdraw the condition that the quantities of the factors other than X are rigidly fixed. The problem is now much more complicated. For the question whether the aggregate remuneration of factor Y will be increased or diminished no longer depends merely on whether its rate of remuneration, *i.e.* its marginal productivity, is increased or diminished. It is no longer, in other words, simply a question of whether its elasticity of partial marginal productivity in respect of changes in X, namely, ${}_x\epsilon_y$ is positive or negative. The issue depends on what happens to the quantity of Y as well as what happens to its rate of remuneration.

§ 10. In § 8 it was shown that, when the quantities of all the factors other than X are rigidly fixed, the aggregate remuneration of any particular factor Y will be increased or diminished according as ${}_x\epsilon_y$ is positive or negative. When these quantities are not fixed, all the ϵ's and also all the e's (other than those affecting X) help to determine the changes in quantity of the several factors, and so come into play. They operate variously in accordance with the manner of their interrelations. We cannot set them out in an inventory, saying, for example, that, the larger is this or the smaller is that, the more likely it is that the aggregate remuneration of factor Y will be increased. For this or that, or, more properly, the influences which govern this or that,

[1] For some purposes it may be convenient to define factor R as being a *complement* to factor Q when ${}_r\epsilon_q$ is positive and a *substitute* for it when ${}_r\epsilon_q$ is negative; and factor Q as a complement to factor R when ${}_q\epsilon_r$ is positive and a substitute in the contrary case. With these definitions, when there are only two factors X and Y and when there are no economies or diseconomies of large scale, X must be a complement, and cannot be a substitute, for Y; and similarly Y must be a complement, and cannot be a substitute, for X. When there are more than two factors, X may be either a complement or a substitute for Y, and Y may be either a complement or a substitute for X; the X – Y and the Y – X relation not necessarily being the same. It may also happen that $1/{}_x\epsilon_y$ has a nil value, in which case we may say that X is neutral to Y. In like manner, of course, Y may be neutral to X. These definitions of complementariness and substitutability between factors of production are not, the reader will observe, on all fours with the definitions of these relations between consumable commodities, as given in Chapter VIII. § 7 and Appendix II.

are favourable or adverse, not absolutely, but according to the way in which they are related to other things. To translate the mathematics into English would not help us. No simple conclusions of a general kind are available.

§ 11. Let us turn to the comparatively simple case in which there are only two factors of production. If the maintenance conditions of X and Y are independent, since, as we have already seen, ${}_x\epsilon_y$ must be positive, provided that the maintenance function of Y is such that a larger rate of remuneration is associated with a larger quantity, *i.e.* that ${}_ye_y$ is positive, the quantity of Y, as well as its rate of remuneration, must be increased. Hence its aggregate remuneration must be increased. But ${}_ye_y$ may be negative. In these conditions it is not *prima facie* necessary for an increase in the quantity of X to entail an increase in the aggregate remuneration of Y. In Appendix XI. §§ 12-13 it is shown that the aggregate remuneration of Y will in fact be increased if its maintenance curve is forward-falling, but not if it is backward-rising. If the maintenance conditions of X and Y are not independent, the existence of a relationship such that the increase in the quantity of X directly entails an increase in the quantity of Y, *i.e.* of a negative value of ${}_xe_y$, enables Y to gain more from an increase in the quantity of X than it would otherwise do, provided that ${}_y\epsilon_y$ is numerically greater than unity. If this elasticity is numerically less than unity the existence of a relationship of that kind is unfavourable to Y, and may entail that an increase in the quantity of X diminishes the aggregate remuneration of Y. The existence of a relationship of the opposite kind has converse effects.

§ 12. So far we have been studying the effect of a shift in the maintenance conditions leading to an increase in the quantity of one factor X, while leaving all the ϵ's and e's unaffected, upon the *absolute* aggregate remunerations of the several factors. I pass to the effect upon the *proportionate* shares of the total product enjoyed by these factors. A full study of this matter would cover the effect upon the proportionate share of X and of *each* one of the other factors, and so indirectly on the proportionate shares of every pair of two factors. Here, however, attention will be confined to

the proportionate share of a factor whose maintenance conditions have been changed as against that of all the other factors together.

§ 13. In the general case, when there are more than two factors and their quantities are not fixed, no simple statement can be made. A mathematical expression of the conditions in which X's proportionate share of the total output will be increased is, indeed, obtainable; but it is exceedingly complex, and not easy to translate into ordinary language. We cannot even provide an inventory of circumstances that make a proportionate gain for X likely, and set them against a second list of circumstances that make it unlikely. For many of the relevant circumstances do not make unambiguously either for proportionate gain or for proportionate loss, but may make in either direction according to what the other circumstances are. All that can be said in general is that the chances of a relative gain for X are better the larger numerically is ${}_x\epsilon_x$. If, while leaving more than two factors of production, we introduce the proviso that the quantities of all of them, other than X, are fixed, the formula that gives the condition for X's relative gain is greatly simplified. It is shown in Appendix XI. § 15 that the occurrence of such relative gain is more probable, not only the larger numerically is ${}_x\epsilon_x$, but also the smaller numerically, if positive, and the larger, if negative, are ${}_x\epsilon_y$, ${}_x\epsilon_z$ and so on.

§ 14. With only two factors of production, provided that their maintenance conditions are independent, it is also shown in Appendix XI. § 16 that, if ${}_ye_y$ is positive or is negative on a backward-rising curve, X's proportionate share of the product will be increased as its quantity increases, provided that the sum of the reciprocals of X's elasticity of partial marginal productivity in respect of itself and Y's elasticity of partial marginal productivity in respect of itself, namely $\left\{\frac{1}{{}_x\epsilon_x} + \frac{1}{{}_y\epsilon_y}\right\}$, is numerically greater than unity. If ${}_ye_y$ is negative on a forward-falling curve this condition is reversed. If the maintenance conditions of X and Y are interdependent, that makes no difference so long as ${}_xe_y$ and ${}_ye_y$ have the same signs or, their signs being opposite, ${}_ye_y$ is numerically less than ${}_xe_y$. If

they have opposite signs, and ${}_ye_y$ is numerically greater than ${}_xe_y$, all the results set out above are reversed.[1]

§ 15. Up to this point we have been studying the implications of shifts in maintenance conditions leading to an increase in the quantity of one factor of production, when the form of the productivity function is given. I turn now to changes in the form of the productivity function when the maintenance functions of the several factors are given. We have to investigate the reactions brought about by such changes on the remunerations of the several factors. The problem has two stages: the simple stage, in which the quantities of all the factors are supposed to be fixed, and the complex stage, where this condition does not hold. At neither stage is it necessary to discuss the effect produced on the proportionate shares of the several factors. Obviously this may be anything whatever. No presumption can be framed about it and no general statement made. Attention, therefore, will be confined to the absolute shares of the several factors.

§ 16. When the quantities of all the factors are fixed, it is evident that the difference made severally to their remunerations depends only on the differences between the forms of the new and the old productivity functions in relation to the existing stocks of the several factors. Let us suppose, to fix the ideas, that productive technique and so on have altered between two periods in the sense that in the new equilibrium the total output of the existing assembly of factors is larger than it was in the old one. There is then some presumption that the absolute shares of all the factors are increased—a presumption that is stronger the larger is the enhancement of total output. Thus, if all we know is that total output is larger than before, we may say that the absolute remuneration of any factor taken at random is more likely to be increased than diminished. But the marginal product of the existing

[1] I have shown elsewhere that the sum $\left\{\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right\}$ is identical, in the conditions here supposed, with the "elasticity of substitution between X and Y" of Dr. Hicks and other recent writers (cf. "The Elasticity of Substitution", *Economic Journal*, June 1934).

quantity, and so the absolute remuneration, of certain particular sub-factors, manual labour of a particular class, or brain labour of a particular class, or equipment or land of a particular class, may easily be diminished. It is even possible that labour as a whole, or capital equipment as a whole, or land as a whole, will be affected unfavourably by a change in the productivity function that renders the total output of these groups of factors in combination larger than before. If any factor Q is affected unfavourably in this sense, this implies that, should that factor insist on being paid the same real price per unit as before, some of it will lose employment. If we choose, we may call a change in the productivity function that is related to Q in this manner Q-extruding; one that is related to it in the opposite manner Q-invoking. But this is merely to coin a name, not to explain anything. In what way any given alteration in the productivity function is related to the remuneration of any particular factor is pure matter of fact.

§ 17. When the condition that the quantities of all the factors are fixed is withdrawn, more difficult issues arise. It is essential that the full complexity of the situation should be realised. The non-mathematical reader may be inclined to imagine that the productivity function generates a set of separate demand curves for the factors, all hanging together, as it were, from a common hook. If this picture represented the facts, a change in the productivity function would merely entail the displacement of some or all of these demand curves. But the picture does not represent the facts, for the reason that the state of demand for any one factor depends in part upon the quantities of the other factors. A transformation of the productivity function from $F(x, y, z)$ to $\phi(x, y, z)$ does not, therefore, imply a specific shifting about of certain demand curves. On the contrary, the way in which one demand curve shifts depends upon how the quantities of the other factors alter; which means that the total change on the demand side cannot be known until we know, not merely that the productivity function has changed from one form to another form, but also what the maintenance conditions are in the face of which this change has been made. In these

circumstances it is obviously possible for a change in productivity of the kind described in the preceding section to lead to a reduction in the remunerations of some factors when the quantities of these are not, no less than when they are, fixed. But whether it will in fact have this effect on any particular factor depends now, not merely on the relation between the new productivity function and the old, but also on the scheme of maintenance conditions.

§ 18. For the purposes of illustration let us imagine that the general productivity function undergoes a change of such a sort that, if the quantities of all the factors remained unaltered, some increase in the marginal productivity of each of them would take place. If these quantities do in fact remain unaltered, this change implies, of course, an increase in the absolute remuneration of each of them. But, with factors whose quantities are free to alter, that implication does not hold. If the maintenance functions of all the factors are independent of one another, and if we ignore the possibility of backward-rising maintenance curves, the quantities of all of them must be increased. For some of them, the increase in quantity, in spite of the fact that it is accompanied by an increase in the quantities of the others, may entail a reduction in marginal product below what it was originally. But, since it cannot be supposed that larger quantities will be maintained in the face of smaller aggregate remunerations, we may say that the aggregate remuneration of every one of them is certain to be increased. If, however, the maintenance conditions of the factors are interdependent, even apart from backward-rising maintenance curves, the quantities of some of them may decline; and this decline may be associated with a fall in the aggregate remuneration, may be of themselves, may be of some other factors. For example, the invention of a new kind of machine that co-operates effectively with particular types of labour may indirectly cause the stock of other kinds of labour to decrease so far that, despite an increase in wage rate per head, their aggregate earnings are reduced.

§ 19. We thus see that types of change in the productivity function, which would increase the absolute remuneration

of each of the factors if their quantities were fixed, *may* diminish the absolute remunerations of some of them if their quantities are variable. *A fortiori* this is true of types of improvement that would diminish the absolute remunerations of some factors even if their quantities were fixed. Thus no simple statement of a general kind can be made. When all the relevant facts about productivity and maintenance are given, what will happen to the absolute share of any given factor in consequence of a given change in the productivity function can be determined mathematically. But it is impossible to derive from the mathematics any broad results that can be expressed in general terms.

CHAPTER XXXV

A ONE-COMMODITY COMMUNITY AND LOCATION OF PRODUCTION

§ 1. In the preceding chapters we have ignored the fact that the world is extended in space. We have implied that factors of production can be assembled together in any required quantities at single centres; and we have assumed that, for all factors of production, there are no costs of movement between these centres. In these circumstances we find (1) that each several factor of production will be paid at each centre the equivalent of its marginal private product of cake there, (2) that the marginal private product of each factor will be the same at all centres, (3) that the several different factors will be grouped together in such wise that the same proportionate quantity of each is contained in each group, (4) that the size of each group and, therefore, the total number of groups, will be such as to make marginal private product and average product everywhere equal, and so, the quantities of the factors being given, to make aggregate output a maximum, and (5), as an obvious rider on the foregoing, that the places at which the several centres are located will be indeterminate—the sport of "accidental" events in the past. We have now to take into account the fact that, for one factor, land in the literal sense, an essential quality is extension in space; that it is impossible to apply varying amounts of it at a point, but, on the contrary, as varying amounts of it are applied, the amount of space covered is proportionately expanded; and that, for practical purposes, land is incapable of being moved at any cost, however high. In these conditions we have to ask, first, in

what way the conclusions previously reached need to be modified when the assumption is retained that the factors other than land can move with complete freedom; and, secondly, what further modification is needed when the fact that the movement of these factors entails cost is brought into account. I shall still postulate, till the last section of the chapter is reached, that all the centres of production are of optimum size.

§ 2. When the factors other than land can move with complete freedom, the distribution in space of each one of them must be such that its marginal private product is everywhere the same. On the assumption, then, that land is a simple uniformly constructed factor of production,—more complex cases can readily be worked out on the lines of Chapter XXV. § 3—such that the qualities of every separate piece of it for the purpose of cake-making are identical, all the centres must contain the same quantity of every factor; and the marginal private products of the factor-groups, in which the several factors are joined, must be equal everywhere to the average products. This condition determines how much of each factor (including how large an area of land) will be included in each centre. *Prima facie* it would seem that, the quantities of the several factors being given, all that there is of all of them must be utilised—which implies that all the land will be occupied; the only issue being the size of the centres held under separate controls. This, however, is not so. If population is sparse, it may well be that some of the land is "redundant", in such wise that the other factors, by combining in the optimum manner with a part of it, will secure a larger product of cake than they would do by combining with the whole of it. In these circumstances a part of the land will be unoccupied; and the marginal private product, and so the rate of pay, of the parts that are occupied will be nil. Whether this is so or not, unless the centres collectively occupy the whole of the earth's surface, the way in which they are distributed over it will, so far as the given data are concerned, be indeterminate.

§ 3. In the preceding section we have taken the extreme assumption that the factors other than land are free to move

without any cost at all. The opposite extreme assumption is that they are totally incapable of movement, or that the cost of movement to them is infinite. We need not, indeed, force imagination to the point of supposing that men and instruments are chained down, each to a single mathematical point in space. It is enough to divide the world up mentally into a number of fairly small finite areas, inside each of which factors of production move freely, but between which they cannot move at all. In these conditions the happenings in each area will be entirely independent of the happenings in any other. Each of them separately obeys the rules laid down in Chapter XXXIII. Each of them can and should be studied separately. The relation between the marginal private products, and so the rates of pay, for example, of any particular factor in any two of them is a secondary consequence of the independent forces that govern each of the areas separately. The more nearly similar these forces are, the more nearly alike the quantities of the factors and the rates of pay in any two areas will be. But there are no interactions between the two, and, therefore, no call for any study of interactions.

§ 4. For an approximation to actual conditions we must, of course, recognise that the cost of movement for factors of production other than land is neither nil nor infinite, but is something between the two. Hence the model of Chapter XXXIII., as qualified by §§ 2-3 of this chapter, is neither a model of the world as a whole nor of any separate isolated part of it. A new element is present of which no account has so far been taken. What has been said above still holds good of each of the several small areas into which we have imagined the world divided: but there is also superadded a certain relation between these areas. Remembering that we are concerned with a stationary state, we can best express this relation in a negative way. If, for each several factor, the difference between its marginal private products, and so rates of pay, in any two areas, when the adjustments proper to each of them separately have been made, is less, relatively to the cost of moving a unit of the factor from the one to the other, than is required to make movement profitable, the situation is exactly the same as it would be with

infinite costs. But, if the relation between difference in rates of pay and costs of movement is such that movement would be profitable, movement will in fact take place. Therefore in a stationary state it is necessary for equilibrium that the factors other than land shall be so distributed among the several areas that costs of movement are everywhere too large relatively to differences in rates of pay to make movement worth while.

§ 5. In a general way the significance of this requirement is plain enough. But its interpretation in detail needs care. Thus consider first the factor labour as embodied in men and women. This factor receives payment at intervals of so much per week or month: while costs of movement, if incurred at all, are incurred as a single lump sum. What lump sum cost of movement balances a given difference in rates of pay depends, therefore, partly on the length of time for which the difference in rates of pay affects the particular units of labour whose movement is in question, and partly on the rate at which these units, or whoever controls them, discount the future. But this does not exhaust the problem. Since we are contemplating a stationary state, we must suppose that the stocks of labour standing in any region are always the same. Hence it would seem *prima facie* that it is the rate of interest *in perpetuo* on the cost of moving a unit of labour from one region to another that the difference between the rates of pay in the two regions must not exceed. But the decision to move or not to move is made by a man whose individual life is limited and who does not perhaps pay great regard to the effect of his action on his successors. Hence what comes into the balance is a substantially larger annual sum than this rate of interest *in perpetuo*; namely, that annuity for the probable life of the man after movement which is equal in present value to the cost of movement. Hence substantially larger differences in rates of pay between different regions may persist without evoking movement than might at first sight be thought admissible.

§ 6. Anybody thinking of capital in its concrete manifestation as capital equipment may be tempted to apply to it reasoning analogous to the above. This would be incorrect.

The cost of moving such and such a quantity of capital, measured in cake, from A to B is not simply the cost of transporting machines of that value from the one place to the other. On the contrary, the cost of moving capital need not involve any costs of physical transport. Thus, if A, who has been accustomed to draw annually from B such and such a quantity of cake as interest on a past loan, decides, one year, to lend this cake to B, the cake that is *not* moved represents a transfer of capital. Such a transfer plainly involves a *reduction* in the aggregate costs of physical transportation. The essential costs of transporting capital from A to B are then the risks that an investor in A suffers from holding his investment in B rather than in A, any special taxes that he may have to pay on foreign investment, and any excess of total income tax that he may have to pay on account of residing in one country and deriving income from another. These are conveniently represented as so much excess annual cost of holding a unit of capital in B as against A. If no movement is to take place, it is this excess annual cost that must be greater than the excess of the rate of interest in cake obtainable from investment in B as against investment in A.

§ 7. The broad effect of this discussion is to show that, when there are nil costs of movement for factors of production other than land, their location at any time, when their quantity is given, is determined wholly without regard to the places at which the several units of them came to birth. At the other extreme, where no movement can be accomplished at any finite cost, the location of the factors is determined wholly *with* regard to these places. In the middle conditions that actually confront us the places of birth of the factors and interactions between these places both play a part. The interactions set limits to the differences in marginal private product, and so in rates of pay, that can rule between different regions without evoking movement. Whatever the productivity functions and the maintenance conditions proper to the several regions separately, it is impossible for the location of factors to be such that these limits are exceeded. Subject to these limits, it is the conditions peculiar

to the several regions by which the location of the factors contained in each of them is governed. Interaction makes certain arrangements impossible, but it cannot, of itself alone, make any arrangement actual.

§ 8. To complete the picture we have to inquire, desires for cake, aversions from work, productive technique and so on being given, in what way the stationary state proper to a higher set of transport costs will differ, after the transition has been accomplished, from the stationary state proper to a lower set. It may happen, of course, that the two states do not differ at all; for conditions may be such that, not only the higher set of costs, but also the lower set is incompatible with any movement. Suppose, however, that in fact the substitution of a particular lower set for a particular higher one entails some movement; and that, in consequence, the second stationary state is different from the first. The general nature of the difference may then be adequately set out if we represent all the factors other than land by a single factor X. Let the fall of costs entail a movement of x units of factor X from region A to region B. Postulate, for a first stage, that the total stock of the factor X that is maintained (and is at work) is unchanged. With the help of the standing environment the transferred units of factor X produce more output than before, while all the other units of it produce the same quantity of output as before. It is certain, therefore, that in the new stationary state *aggregate* production will be larger than in the old. The consequences to the manner of distribution of the product between X and the other factor, land, are also readily found. If the elasticity of total marginal productivity of X in A over the relevant range is (numerically) less than unity and the elasticity of total marginal productivity of X in B is (numerically) greater than unity, X in the new state will receive larger aggregate remuneration than before in both A and B: in converse conditions smaller aggregate remuneration in both. In the former case, therefore, the total remuneration of X must be larger in the new state, in the latter smaller. If the elasticities in both A and B are either greater than unity or less than unity, the total remuneration of X may be either larger or smaller in the new state

according to the detailed circumstances. The total remuneration of the other factor, land, is necessarily increased if that of the factor X is diminished, and may be increased even though the total remuneration of that factor is increased.

§ 9. So far we have assumed that the aggregate quantity of the representative factor X, that is maintained and at work, is the same when costs of movement are high as when they are low. This need not and, in general, will not be so. It is to be expected that, in the region where the rate of pay per unit of this factor is made higher by the establishment of a lower cost of movement, more of it than before will be originated; though it is *possible* that less will be originated. In the other region converse conditions rule. It is, therefore, not certain whether the aggregate quantity of factor X in the two regions together will be larger or smaller when transport costs are lower than when they are higher. Though, therefore, the conclusions, that were reached in the preceding section on the assumption that the aggregate quantity of factor X is fixed, may still be set down as probable, now that that assumption is removed, they are infected with an element of doubt.

§ 10. In the foregoing discussion, while it has not been necessary to assume that economies and diseconomies of large scale are absent, the less stringent postulate, that each region, into which our community is separated by costs, is large enough to allow the centres of production to be of optimum size has been utilised. This postulate may not be justified. It is obviously less likely to be justified with the community divided up into regions between which the movement of factors entails costs than it would be if these costs were nil. If it is not justified in any region, the situation described in Chapter XXVI. § 2 will be established. While the factors other than the hiring factor will still be paid at a rate equal to their marginal private products, the residue left over for the hiring factor may be either too large or too small to allow of its rate of pay being equal to its marginal private product.

CHAPTER XXXVI

TRANSITION TO A MANY-COMMODITY COMMUNITY

§ 1. IN Chapters VIII.-X. we studied a Crusoe Economy containing one person and many commodities; in Chapters XVI.-XXI. an economy in which there were many persons and many commodities, but each person made independently the service or finished article which he had to sell, and did not co-operate either with other people or with non-human factors of production, other than free goods, in his work. We then turned, in Chapters XXII.-XXXV., to a model system in which many different sorts of persons and many different sorts of non-human factors co-operated in production, but only a single sort of commodity was being produced. The time has now come so to modify this last model as to bring into account the existence of many different sorts of commodities. The new model is distinguished from the old in that interchange of commodities is now present, and, therewith, the need for factors of production to be distributed over the manufacture of a number of different articles. From this primary difference there emerge a number of secondary differences of great practical importance. As a prelude to the more detailed analysis of the Chapters that follow it will be convenient to set them out here in a summary way.

§ 2. The first and most far-reaching complication is on the maintenance side. Its significance, indeed its existence, is obscured, for any but careful students, by our habit of speaking of the maintenance price of such and such a quantity of any factor in terms of money. We are apt to forget that rates of pay in money only have meaning when the prices of the things, which the people embodying or owning the several

factors of production wish to buy with their pay, are given. When these prices are liable to vary relatively to one another, it is no longer a question of different amounts of a *defined entity* being offered to any factor and responded to by it in accordance with an unambiguous elasticity of maintenance. It follows that the reactions on the side of maintenance cannot now be expressed in any simple way. They depend in a highly complex fashion on the tastes and purchasing power of the persons who control pieces of the several factors of production. In view of this, when a change on the demand side occurs, though the relation between the new situation and the old can be formulated in a determinate system of mathematical equations, no conclusion of economic interest can be educed from them. We must, therefore, perforce rest content with a partial and limited discussion, whose aim is merely to illustrate, not to exhaust, its subject-matter. To this end I shall assume, for the main part of the following chapters, that the quantity of every factor is rigid against disturbances through secondary reactions, *i.e.* that the elasticities of maintenance of all the factors are nil. Of course, in fact this condition is not satisfied. But by making the assumption that it is satisfied we are able to bring out more clearly than would otherwise be possible a number of essential features of a many-commodity community, which are not found in a one-commodity community.

§ 3. Secondly, the man who sits, so to speak, in the hub of the economic wheel, performing, in the manner described in Chapter XIII. § 3, the function of hirer, in a many-commodity community performs also, along with this, a second associated function. He sells in the market the commodity produced in the centre to which he belongs. If this commodity were divisible into sufficiently small parts, he might, indeed, as was indicated in Chapter XIV. § 2, pay it out in kind to the factors that he hires, leaving it to them to exchange what he hands to them for the things in which they wish to take their income. But, as was also indicated in the chapter cited, in some industries, *e.g.* those that make Atlantic liners, this type of arrangement is impossible; and in any event it is cumbrous and inconvenient. In practice, therefore, the hirer always

pays the factors that he engages with money, and sells against money the commodity that they help to make. He is thus not, as in a one-commodity community, a simple hirer of factors, but a hirer of factors *plus* a seller of commodities. In a many-commodity community, in short, in place of the hirers of a one-commodity community there stand what we may call hirer-sellers. With all the centres of production of optimum size and everybody acting on the rule of competition, the seat of the hirer-seller is an unprivileged seat, winning for its occupant no special remuneration, precisely on all fours with the seat of a simple hirer as described in Chapter XXVII. § 6. But, as will appear presently, when the rule of monopoly is followed, the hirer-seller has opportunities for gain not open to the simple hirer.

§ 4. Thirdly, even though conditions are such that (1) all the centres of production are of optimum size and (2) there are no available external or external-internal economies or diseconomies, multiplicity of commodities entails the presence of a number of new variables. This fact is responsible for several complications that were not found in a one-commodity community. An attempt is made to elucidate these matters, subject to the reservation of the last section, in Chapters XXXVII.-XXXIX.

§ 5. Fourthly, the validity of the above two conditions comes into question. In a one-commodity community, whose parts are not separated by any costs of transport, it was held that both of them could properly be postulated. The ground for this was the enormous aggregate output of a single commodity that must be supposed to take place if our one-commodity community is adequately to represent the actual world. No doubt, anyone who confines his attention to the broad statistics of production in a highly developed country such as England—to say nothing of the world as a whole—may be tempted to conclude that most important commodities are produced in quantities so large that both our two conditions will still hold good. But this is a shallow view. It is, of course, true that a highly developed region is likely to produce in very large quantities most of the sorts of commodities that are produced at all, *provided that* the term "sort of com-

modity" is given its loose popular signification. But a single sort of commodity in this sense usually embraces a great number of different individual types. Thus, to take an extreme case, for purposes of popular classification "machinery" is a single sort of commodity; and in modern conditions all large industrial countries produce an enormous amount of machinery. But within the general class machinery there are an immense number of different individual machines, and of some of these only a very small annual supply is needed by the whole of the world. For example, there is a certain instrument used in some classes of engineering, for which, before our recent tariff legislation, all the engineering concerns of this country relied on a single highly specialised German firm. The postulates, which we rightly adopted for the one representative commodity of a one-commodity community not broken up by costs of transport, are, therefore, often not appropriate to actual commodities in a many-commodity world. When factors of production are held apart in different places by transport costs, we have seen that, even in a one-commodity community, they may often fail. In a many-commodity community of equal size failure is still more probable. The second postulate, no external or external-internal economies—we may disregard possible diseconomies—will plainly fail over a wide field; the first, optimum size of centres of production, even though everybody acts in accordance with the rule of competition, will fail in respect at all events of some industries.

§ 6. The fifth respect in which a many-commodity community differs from a one-commodity community has to do with the opportunities for, and consequences of, action in accordance with the rule of monopoly. To elucidate this matter we need to develop briefly certain broad implications of the analysis conducted in Chapter XVI. The hirer-seller in control of any centre of production, if he realises exactly what his own interest is and acts so as to promote it, will determine the output of his centre in such a way as to maximise his net receipts, *i.e.* the excess of the total sales price of his product over the expenses of hiring, at the price ruling elsewhere, himself and the other factors of production

that collaborate with him. That is to say, he will not accept the sales price of the market, but will have regard, in determining his output, to the effect of his own conduct on that price. He will thus in all circumstances act, in principle, on the rule of monopoly. If, however, the demand for deliveries of the product from him is perfectly elastic over the relevant range, the output by which his net receipts are maximised will be that output which makes demand price equal to his marginal expenses of production, *i.e.* to the sum of the values of the quantities of the several factors, including himself, engaged on the marginal unit of product. This coincides with the output that he would produce if he followed the rule of competition. There are two conditions, either of which makes the demand for deliveries from him perfectly elastic. The first is that throughout the community only one sort of commodity is being produced, so that the demand for it is in terms of itself. In this case the demand for the whole body of deliveries of the commodity from all the centres together must, in the nature of things, be perfectly elastic: *i.e.* the price offered must be the same irrespective of the quantity. Hence, *a fortiori*, the demand for deliveries of the commodity from any one centre must be perfectly elastic. The second condition is that the deliveries of the commodity from the one centre constitute a very small part of the total deliveries. In this case, even though the demand for deliveries as a whole is highly inelastic, the demand for deliveries from the one centre must—provided that costs of transport of the commodity, in the wide sense to be defined in § 7, are nil—be perfectly elastic; or, more strictly, as the proportionate share of the product due to the one centre becomes smaller and smaller, must approach to perfect elasticity without limit. In both these cases the consequences of monopolistic action are identical with the consequences of action in accordance with the rule of competition; so that it does not make any difference which of the two rules is followed. In all other cases it does make a difference. If the demand for deliveries of the product from any centre is not perfectly elastic over the relevant range, the output which maximises that centre's net receipts will be less than the output proper to

competition. Since, therefore, neither of the two conditions distinguished above is likely, in a many-commodity community, ever to be realised fully, when the rule of monopoly is followed, that fact has more than a merely formal significance. The fact, however, that the hirer-seller in control of any centre of production, *if he realised what his own interest is and acted on it*, would follow the rule of monopoly, does not imply that he in fact follows this rule. It was argued in Chapter XVI. that people do not exactly realise what their own interest is, but ignore altogether elements below a certain size. For this reason the hirer-seller, who controls a small part of the output of any commodity, though, since this part is not literally infinitesimal, he would gain *something* by acting monopolistically, will not in fact do this, but will accept the price of the market without attempting to influence it. How small his share of output must be for this to happen cannot, as we saw in Chapter XVI., be ascertained by ratiocination, but is brute fact. Experience suggests, however, that it need not be very small. In an industry, therefore, where the optimum size of individual firms is moderately small in relation to the total demand of the market, so that the number of separate centres is fairly large, these centres, provided they do not combine, may be expected to act, not monopolistically, but in accordance with the rule of competition. Thus in a many-commodity community, not divided into separate parts by costs of transport for factors of production, the rule of competition will probably prevail for a substantial number of industries.

§ 7. The study of a many-commodity community, upon which, with these preliminaries, we have now to embark, like the preceding study of a one-commodity community, is most conveniently taken in stages. First, we suppose that there are no costs of transport of any kind, either for factors of production or for commodities. Moreover, absence of costs of transport is taken to carry with it absence of all such incidents as custom-engendered preference, on the part of particular purchasers of any commodity, for dealing with any one seller rather than any other—preferences that act in the same way, and whose effects can be analysed on the

same plan, as costs of transport proper. This assumption is carried through Chapters XXXVII.-XLII. In Chapters XLIII.-XLV., the costs of transport for factors of production still being assumed to be nil, costs of transport for commodities are brought into account. They are shown to make it possible for several monopolistic sellers to co-exist in the same industry without the situation being rendered indeterminate. In Chapter XLVI., which plays for a many-commodity community a part complementary to that played by Chapter XXXV. for a one-commodity community, some consequences, that follow from various assumptions about costs of transport for commodities when factors of production are imperfectly mobile, are studied: and in Chapter XLVII. the discussion of our fourth and final model, which we are now beginning, is rounded off with a brief comment upon the consequences to production and distribution of various sorts of State action.

CHAPTER XXXVII

THE CONDITIONS OF EQUILIBRIUM IN A MANY-COMMODITY COMMUNITY IN WHICH THE RULE OF COMPETITION PREVAILS EVERYWHERE

§ 1. For the first stage of this programme let us imagine a many-commodity community in which the rule of competition prevails everywhere. In such a community it is not *necessary* that all the centres of production shall be of optimum size. Situations are possible in which in some, or, indeed, in all, industries men of given type acting as hirer-sellers are earning more than similar men acting as managers, and yet nobody is induced to shift from the manager function to the hirer-seller function, because, if he did, and in doing so, reduced the number of centres by one, in the new situation he would earn *less* than similar men acting as managers; and conversely. Situations of this type are obviously compatible with all-round obedience to the rule of competition; and they imply *some* departure from optimum size. But, as we saw above, that rule is not likely to be followed in any individual centre unless departure from it would yield only a small gain; unless, that is to say, the industry to which it belongs is so large that the centres of production there approach very nearly to optimum size. Hence, when the rule of competition prevails everywhere, though some departures from optimum size may, indeed must, occur, the departures will be too slight to matter. The centres will always be so near to optimum size that we can without appreciable error regard them as attaining exactly to that size. Hence in this and the two following chapters, where universal prevalence of the rule of competition is postulated, this postu-

late will be taken as carrying with it the further postulate that all centres of production are of optimum size. Since, as was premised in § 7 of the last chapter, we are also postulating that no cost of transport impedes the movement of factors of production, the state of affairs we are contemplating is substantially that conceived by classical theory under the name of free competition. In this type of many-commodity community the demand price, which is also the actual rate of pay, of every factor—including the class of persons from whom the heads of businesses are drawn—is equal, wherever it is employed, to the value there of its marginal private product; and this value is, for each factor, the same in all industries. Thus the marginal private product of, say, labour, or some class of labour, in the bicycle-making industry must consist of a number of bicycles that will exchange against, or have the same value as, the number of hats contained in the marginal private product of the same factor in the hat-making industry. In short, in a many-commodity community of the type here imagined the picture we have drawn in earlier chapters of a one-commodity community broadly represents the facts, save only that here *value of* marginal private product plays the part that was there played by marginal private product *simpliciter*.

§ 2. These results imply a particular systematic arrangement of the factors of production. In a one-commodity community, the quantities of the several factors being given, they all engage in producing the one commodity: and they are divided up in equal quantities among a collection of similarly constituted centres, the number of which is fixed by the condition that in each of them the marginal private product of the factor-group units engaged there is equal to the average product. In a many-commodity community the situation is much more complex. To disentangle the various influences at work it is convenient once more to proceed in two stages. In the following section I shall imagine that, while many commodities are being produced, there is only one kind of factor of production. Having studied this simplified case, I shall then in § 4 bring into account the fact that in real life there are several kinds of factors.

§ 3. With one factor of production but several commodities the distribution into centres of such quantity of factor as is engaged on any one commodity is determined, this quantity being given, in exactly the same way as though there were only one kind of commodity in existence. But there is now also to be determined the distribution of the factors between different commodities. This clearly depends partly on the average, which is the same as the marginal, private productivity of the factor in respect of different commodities; but also partly on people's tastes. Nor is this all. It is not tastes by themselves, but tastes in conjunction with purchasing power, that are significant here; and the purchasing power of different people varies directly with the number of units of the factor that they individually control. Thus, besides the productivity of the factor in respect of different commodities and the taste functions of different individuals, there is also relevant the manner in which tastes for different things are distributed among people embodying or controlling different quantities of the factor. We need to know which taste function is bound up in the same individual with which degree of productive capacity. This statement is perfectly general and is applicable alike to a barter economy and to a money economy. For the latter type of economy it may be expressed by saying that the money demand system of the community is made up from a combination of the money demand systems of its members; that the money demand system of each member depends both on his taste system and his individual money income; and that his individual money income depends on the kind and quantity of productive capacity that he individually possesses together with the tastes and purchasing powers of the other members.

§ 4. When account is taken of the fact that there are several factors of production, and not merely one, it is no longer true with many commodities, as it was with only one commodity, that the several factors are combined in each centre in the same proportions, namely, the proportions in which they severally exist. On the contrary, the factors will, in general, be allocated in different proportions in the centres belonging to different industries. The allocation is determined

in a system by the mutual play of all the demand conditions in conjunction with the state of productive technique ruling everywhere. Obviously, for example, the proportion of land, as compared with labour, will be larger in wheat-growing than in lace-making. But the proportions of the factors in the centres belonging to any one industry are not determined by the circumstances of that industry alone. On the contrary, the proportions in each industry are determined in a complex system by the circumstances of all.

§ 5. What has been said, it will be noticed, entails that, for purposes of comparison between centres producing different commodities, the concept of size is no longer, as it was when one factor of production only existed, unambiguous. Comparisons may, of course, be made, if so desired, in terms of the money values of the factors at work in different centres. But this is a conventional, not a direct physical comparison, such as was possible in simpler conditions. Since the units of the several factors are not physically commensurable, physical comparisons of size can only be made between two centres when in one of them there are more units of some factor and not less units of any factor than in the other.

CHAPTER XXXVIII

THE REMUNERATIONS OF FACTORS OF PRODUCTION IN TWO STATIONARY STATES UNDER THE RULE OF COMPETITION, IN ONE OF WHICH THE QUANTITY OF ONE FACTOR IS LARGER THAN IN THE OTHER

§ 1. I PROPOSE next in this and the following chapter to conduct, for a many-commodity community, in which the rule of competition prevails everywhere, all the centres of production are of optimum size, and there are no economies or diseconomies of large scale, an inquiry analogous to that conducted for a one-commodity community in Chapter XXXIV., but simplified by the assumption that the maintenance functions of all the relevant factors and sub-factors are absolutely inelastic. In this chapter I shall discuss the effects on the absolute remunerations[1] of the several factors of changes in the quantity of one of them. The problem will be examined in §§ 2-3 on the assumption that the proportions in which people purchase with their money income various sorts of commodities, though they may be different after the change from what they were before, alike before and after the change are the same for everybody. In §§ 4-6 the consequences of dropping that assumption will be examined.

§ 2. Let us consider first the effect of a change—say an

[1] When there are a number of commodities, questions about the *proportionate* shares, in any physical sense, secured by different factors are plainly nonsense questions, except in the special case of factors whose purchases of all commodities are in the same proportions. But questions about proportionate shares in terms of *value* are, in principle, capable of being answered; for the proportions of this value enjoyed in any situation by any factor is the same, no matter in terms of what commodity the valuation is made.

increase—in the quantity of one factor upon the absolute earnings of that factor itself. It is easy to see that, if in every industry the productivity of this factor in the neighbourhood of the relevant quantities has an elasticity greater than unity, the factor will secure a larger aggregate amount of each several sort of thing than it had before. This implies, on the assumption set out at the end of § 1, that its absolute share of the total output is unambiguously increased. In the converse case, if in every industry the productivity of the factor has an elasticity less than unity, the factor's absolute share will be unambiguously decreased in consequence of an increase in its quantity. In real life it is to be expected that for some industries the productivity of our factor in the neighbourhood of the relevant quantities will have an elasticity less than, for others one greater than, unity. In these circumstances that factor will receive after the change more of some sorts of things than before and less of other sorts. We are then not able to say unambiguously whether it will receive larger or smaller real earnings.

§ 3. Turn next to the effect on the absolute earnings of the other factors of an increase in the quantity of one factor, tastes and everything else remaining, as before, unchanged. Evidently the additional stock of the one factor will spread itself over the several industries. It is impossible that a smaller amount of it can be engaged in any industry than used to be engaged there, and, except in the limiting case of industries in which either more factors would yield no product or more product would find no market, a larger amount than before will be engaged in every industry. It follows that in every industry the output is larger than before. But, in accordance with the argument of Chapter XXXIV. § 8, not all of the extra output is absorbed by the expanded factor. Hence the other factors collectively get more than before of the commodities produced in every industry. On the strength of this we are entitled to say that their absolute share of output, or real income as a whole, is definitely and unambiguously increased. The fact that output is made up of a mixture of different things, whose amounts have almost certainly increased in different proportions, does not prevent

us from saying this; though it does prevent us from saying, as we could do if only one sort of commodity existed, that their absolute share of output has increased by some definable percentage. The result for the other factors collectively of an increase in the stock of one factor is thus the same in a many-commodity world as it is in a one-commodity world. Moreover, it is easy to see that here, just as in that world, though the other factors in the aggregate must gain, any individual other factor, if there are more than one, need not gain. The conditions determining in what conditions a given individual factor will gain are of the same general character, but much more complex in detail, as those which determine them in a one-commodity world.

§ 4. Let us now remove the assumption that, alike before and after the change in the quantity of our factor, everybody purchases different sorts of commodities in the same proportions. This assumption is, of course, an extravagant one; for obviously poor people do not buy so much champagne relatively to bread as rich people do. Let us now recognise that representative units of different factors of production take out their earnings in differently constituted collections of goods. This implies that after the change in the quantity of our factor, while the constitution of *some* people's collections may be the same as it was before, the constitution of *all* people's collections cannot be the same. Obviously a great number of different arrangements are possible; and it is impracticable to examine them all. Attention will, therefore, be confined to the case in which the constitution of the collection purchased by the factor whose quantity has been increased is the same after the increase as before. I shall inquire in turn how far the conclusions of §§ 2-3 respectively have now to be modified.

§ 5. We have seen that, if the productivity in all industries of the augmented factor has an elasticity greater than unity, that factor, after the increase, is free in each industry to carry off a larger amount of the product made in that industry than it could have carried off before. But this does not necessarily imply that it is free to carry off a larger quantity of the specialised collection on which it in fact

spends its income. Conditions may be such that its enlarged command over things in general does not enable it to buy so much of the things in which it is interested as it was able to buy before. In like manner, even though its productivity has everywhere an elasticity less than unity, it need not necessarily obtain less than before of the collection of things in which it is interested. The conclusions of § 2 are still probable, but in the more general conditions that § 4 brought into account they are no longer certain.

§ 6. The conclusions of § 3 as to the effects of an enlargement of one factor on the absolute share of all the others collectively also suffer damage from these more general conditions. If the enlarged factor secures less than it used to do of each of the things in which it takes out its earnings, the other factors must, indeed, gain unambiguously. For, as we have seen, the output of every kind of commodity must be increased, or, at the worst, not diminished. The other factors collectively must, therefore, secure more of some commodities than before and not less of any. Thus their real income is unambiguously increased. But, if the enlarged factor secures more than it used to do of any of the things in which it takes out its earnings, it may happen that, for some of these things, its extra takings exceed the extra output, so that the other factors collectively are left with less of them than before. They receive then more of some commodities but less of others. In this case there is no unambiguous answer to the question whether they receive larger or smaller real earnings.

CHAPTER XXXIX

THE REMUNERATIONS OF FACTORS OF PRODUCTION IN TWO STATIONARY STATES UNDER THE RULE OF COMPETITION THAT DIFFER IN RESPECT OF TECHNIQUE OR TASTES

§ 1. HAVING considered the consequences, as between two stationary states, of differences in the quantity of one factor of production, all other things being the same, let us turn to the consequences of differences in technique and taste in respect of particular industries. I again proceed in two stages. As before, for the first stage I postulate that, alike before and after the change, the proportions in which the several factors purchase various sorts of commodities are the same; for the second stage I abandon that postulate.

§ 2. Let us begin with an improvement of such a sort that in the industry to which it applies the same proportionate combination of factors of production is still the optimum, but, say, 10 per cent less of every sort of factor is needed for a given output. Provided that initially the several factors were engaged in the same proportion in that industry and in every other industry, there will be no shift in the proportions in which they are employed anywhere. Hence the relative rates of pay per unit of each several factor, in terms of anything we please, and so also their respective proportionate shares of dividend will be unaltered. If the demand for the commodity in respect of which the improvement has been made is such that the quantity of factors engaged in producing it is diminished, the dividend will contain more than before of all sorts of commodities, or, more strictly, more of some and not less of any sorts. It follows that the absolute share of every factor is unambiguously increased. If the

demand for the commodity in respect of which the improvement has been made is such that the quantity of factors engaged in producing it is increased, the dividend must contain less than before of all *other* sorts of commodity. Every factor, therefore, will receive more than before of the commodity whose productivity is improved, but less of other commodities. Even in this case, however, we can assert that its absolute share is increased, if we adopt the device of Chapter VI. § 5, and postulate *by definition* that real income is enlarged by improvements, provided that the quantities of all the factors at work and all other relevant conditions are unchanged.

§ 3. The proviso of the last section that initially the several factors were employed in the improved industry in the same proportions as in every other industry need not be satisfied. To treat this case, let us suppose that there are only two factors A and B. If factor A is engaged in a larger proportion, relatively to B, in the improved industry than elsewhere, the relative money income of A, as against B, will be affected differently according as, in consequence of the improvement, more or less of the appropriate complex of factors is engaged in the improved industry; *i.e.* according as the demand for that industry's product has an elasticity greater or less than unity. In the former event the proportion in which factor A, relatively to B, is engaged in other industries is diminished, so that its relative share of the money value of the dividend is increased. It follows from the results of the last section that its absolute share must be increased; while the absolute share of factor B *may* be diminished. In the latter event, the proportion in which factor A is engaged in the unimproved industry relatively to the other factor is increased, and its relative share of the money value of the dividend diminished. Hence its absolute share *may* be diminished, while the absolute share of B *must* be increased.

§ 4. The above result was obtained upon the assumption that the constitution of the factor-group units engaged in producing each of the several sorts of commodity is the same after the improvement as it was before. But normally this assumption is not warranted, and these proportions will be

altered. This entails an all-round modification alike in the constitution of the factor-group units engaged in every industry and in the relative values of the different factors. Even so, the foregoing analysis is still in substance applicable. The crucial consideration is what happens to the proportion in which the factors are engaged in industries other than the one in which the improvement has taken place. The proportion of factor A relatively to B engaged there is increased if, some units of both factors being drawn into the improved industry, the ratio of A's influx to B's influx is smaller than the ratio of A's quantity to B's quantity initially in other industries, and likewise if, some units of both factors being pushed out of the improved industry, the ratio of A's efflux to B's efflux is larger than the ratio of A's quantity to B's quantity initially in other industries. From this point onwards the analysis is the same as in the preceding section.

§ 5. In both sections equally it has been assumed that there are only two factors, A and B. If there are more than two factors, we can still obtain results on the lines of those set out above, provided that the proportion of some one factor A to any one of the other factors engaged in the unimproved industries is affected by the improvement in the same sense as its proportion to any other of them. The factors other than A are then collectively in the same position as factor B in the two previous sections. But, if there are more than two factors altogether, and the proportion of no one of them engaged in the unimproved industries is affected in the same sense as against everyone of the others, the analysis of §§ 3-4 ceases to be applicable, and no conclusion of the type there reached can be drawn.

§ 6. Let us now remove the assumption that, alike before and after the change, the proportions in which any one factor purchases different sorts of commodities is the same as the proportions in which every other factor purchases them. In this case, even if the proportions in which the several factors are engaged are the same in all industries both before the change and after it, it might seem, on the analogy of Chapter XXXVIII. § 5, that the absolute shares of some of the factors, in terms of the things they severally choose to buy,

may be diminished. Provided, however, that, as we are assuming, there are no economies of large scale anywhere, this inference is wrong. For, the proportions in which the factors are everywhere engaged being unaltered, after the change each factor must be worth per unit the same amount as before of every unimproved commodity and more than before of the improved commodity. It follows that factors which buy any of the improved commodity earn larger absolute remunerations than before in terms of the things in which they are interested; while factors which buy none of the improved commodity earn precisely the same remunerations as before. In more complex cases, however, where the proportions in which the factors are engaged in different industries are different before, and consequently also after, the change, the contention of § 3, that as a consequence of the improvement the absolute remuneration of some factors *may* be diminished, holds good, and is, indeed, reinforced.

§ 7. Let us turn to the consequences of an enhancement of taste for some one commodity, all other things, as before, being supposed, unchanged. This carries with it an increase in the quantity of the commodity primarily affected and in the quantities of the several factors of production engaged in producing it; along with a decrease in the quantities of each of the other commodities and of the factors of production engaged in producing them. The proportions of the factors so engaged on the commodity primarily affected may or may not be altered when the quantity produced is altered. In any event the proportions of the several factors available elsewhere are likely to be altered. With this starting point an analysis on the same lines as that worked out in §§ 3-4 can be carried through. The crucial point is the effect on the proportions of the several factors available elsewhere. The rate of pay in terms of any commodity other than the one primarily affected will be increased for factor A and diminished for each of the other factors if, in the outside industries, the proportion of A's quantity to the quantity of each of the other factors is diminished; and conversely. Since the dividend after the change contains more than before of some things and less of others, the question whether the share of any

factor that takes an aliquot part of each item contained in it is absolutely larger or smaller is, in principle, unanswerable. If we imagine some factor, which consumes different things in the same proportions after the change as it did before, this question about that factor is answerable in principle. But the answer depends on the detailed circumstances and cannot be given in general terms.

§ 8. So far we have imagined a shift of taste taking place unaccompanied by any shift in the maintenance functions of any of the factors. But in real life this is not possible. For the people, whose tastes are supposed to shift, themselves constitute, or control, factors of production. If unskilled labourers develop a passion for cinema shows, so that their standard of life comes to include an abundance of these, this will cause the stock of unskilled labour, that a given environment suffices to maintain, to be smaller than before. Thus shifts of taste have more complex implications than developments of technique. The latter may, and, except in so far as they indirectly bring about shifts of taste, will, entail modifications only in the general *scheme of demand* for factors of production. Shifts of taste, on the other hand, entail, not only these modifications, but also modifications in the maintenance function of one or more of the factors. For a full study of their effects, therefore, the analysis of the preceding section is not sufficient. A combination between it and the analysis of Chapter XXXVIII. is required.

CHAPTER XL

MONOPOLY IN A SINGLE INDUSTRY SUBJECT TO ALL CENTRES OF PRODUCTION BEING OF OPTIMUM SIZE

§ 1. Throughout the last three chapters we have supposed that the rule of competition prevails everywhere. We have now to examine the consequences of substituting monopolistic action for action in accordance with that rule in a single industry, while all the other industries are continuing to act on the rule. I am not concerned here either with several monopolies alongside of one another in the single industry or with one monopoly in it alongside other centres that act on the rule of competition. Attention is confined to the case of one monopoly, whether a combination of centres or a single centre, that covers the whole of the industry in which it is set up. The situation which prevails when a single industry is governed by a monopoly of this kind, all other industries following the rule of competition, is to be compared with that which prevails when *all* industries obey that rule. It is, of course, postulated that the technique of production is exactly the same in both situations; so that the only difference between the two has to do with the selling policy of the single industry. It is postulated further that every commodity is sold in one market, so that no question of price discrimination can arise. Finally, for the first stages of the inquiry it is convenient to suppose, as we have already done on several occasions, that there is only one factor of production, or, what comes to the same thing, that, while there are several factors, they are bound everywhere together in rigid uniform factor-group units.

§ 2. In Chapter XX. §§ 3-4 a study was made of the conse-

quences of monopolistic action on the part of a single representative person, specialised in the production of one commodity and engaged in interchange with a number of other specialists, all of whom act on the rule of competition. It was shown that the monopolist is bound to contract the amount of his work and output below what he would provide if acting on the rule of competition. This would come about through his seeking to make marginal aversion from work equal to desire for marginal value of product, instead of to value of marginal product. With monopoly by a combination of centres of production, if the centres were composed simply of co-operating groups of workpeople, these persons might be expected to take out some of their monopoly gains in reduced hours of work. In actual life monopolistic bodies generally comprise large quantities of capital as well as of labour. Moreover, the head of the combination is free to obtain labour from the outside world, where there is no reason for the length of the working day to be modified. Hence it is very unlikely that any appreciable shortening of hours will take place. That effect may for practical purposes be disregarded. But it is the only channel through which the form of monopolisation contemplated in Chapter XX. operates. The form that concerns us here is thus essentially different from that form, and must be examined from a different angle.

§ 3. I shall here study only monopoly by combination, the number of centres being so large that each of them may be presumed to approximate to optimum size.[1] It is plain that this sort of monopoly cannot, in a stationary state, advantage those who undertake it, and, therefore, will not in fact be undertaken, if new centres are free to establish themselves in the industry. For in the conditions here supposed any centre of optimum size can produce at as low a cost as the combination, and so can prevent it from exacting monopoly prices. The combination, therefore, must be able to estop this freedom. There are a number of ways in which it can do this. Potential competition can be extruded by legal rules

[1] For some comments on monopoly in single-centre industry, cf. Appendix XII. § 3.

conferring on particular groups of people the exclusive right to manufacture some given article; or a particular group may own the only source from which a raw material essential for making its product is obtained; or it may be in a position, by the threat of various sorts of clubbing device—boycott or cut-throat competition—to frighten rivals away. There is thus ample opportunity for monopoly by combination to be introduced and maintained.

§ 4. A distinction may be drawn at the outset between two cases. We have seen that, unless the monopolistic body can prevent outsiders from constituting themselves into centres for making and selling its product, its monopoly will be useless. But the extrusion of outsiders from constituting new centres may or may not carry with it extrusion from attachment to the monopolised industry. If it does not do this, a special kind of waste is set up through equipment and labour being brought into the industry on the strength of the high expectation of earnings there, and then stranded in idleness. Sometimes the waste takes the form of more centres being established than would suffice to provide the output decided on by the monopolist if they were fully worked, and then being only worked partially, *i.e.* below optimum activity. This often happens under price-fixing kartels whose membership is not restricted. It also happens under price-fixing schemes backed by government authority, where there is no control over the establishment of new, or the continuance of old, productive centres. These wastes, however, are not inherent in monopoly by combination. Indeed they are plainly a consequence of defective operation of it. In what follows attention will be confined to those more efficient types, in which extrusion from work in the monopolised industry carries with it extrusion from attachment to the industry.

§ 5. It is evident in a general way that monopoly by combination in any one industry will cause a smaller quantity of factor-group units to be at work in that industry, and so a smaller quantity of the commodity made by it to be produced, than would happen if competition ruled everywhere. The extent of the cut in work and output depends on the form of the demand curve and of the average cost curve

—the cost of a factor-group unit being reckoned at what it could earn outside the monopolised industry—over the whole relevant range; not, of course, merely upon the elasticities of the curves at the end points. It is a familiar fact that, if the curves of demand and of supply (in the sense of full average cost) for any commodity are straight lines, the output proper to monopoly will be half that proper to competition; if both curves are convex towards one another, less than half, if both are concave, more than half. In Appendix XII. § 1 it is shown further that, if both the demand and the supply curves over the relevant range are constant elasticity curves, and if the elasticity of supply is positive and greater than unity, the two curves *must* be convex towards one another. Moreover, the cut in output being given, the cut in factor-group units engaged will obviously be larger if their average product increases with a reduction in their quantity than if it is stationary or decreases. It follows that the cut is larger if external or external-internal economies are than if they are not available to the monopolised industry.[1] In any event there is some cut, and some factor-group units are withdrawn from work in the monopolised industry. With wastes of the kind described in § 4 excluded, all the factor-group units so released enter other industries. Hence, while the output of the monopolised industry is diminished, that of some or all of the other industries is increased. It is, therefore, not possible to say in physical terms whether the aggregate real income of the community is larger or smaller than it would have been under all-round competition.

§ 6. So far we have considered the effect of monopolisation in a single industry upon productivity as a whole. Let us turn to the effect on the earnings of those factor-groups which stand respectively inside and outside that industry. The group outside the industry, on the assumption that it is

[1] It may perhaps be objected that the cut in output, being itself partly dependent on the form of the supply curve, cannot legitimately be taken as given, as is done in the text. This would be correct if the presence of external or external-internal economies had any general tendency to make output contract under monopoly more than it would otherwise do. But there is no such general tendency. With straight-line curves, for example, monopoly halves output irrespective of the inclination of the supply curve.

reasonably small relatively to the sum total of industries, may be presumed to earn per unit, in terms of the product that they themselves make, approximately the same rate of pay as before. For, since, by hypothesis, competition rules, the centres they serve are of optimum size, so that there are no internal economies; while the proportionate change brought about in the scale of any one of them must be too small to allow external or external-internal economies to operate to any appreciable extent. Now after monopolisation a unit of the monopolised product will be worth more than before of all other products. It follows that the factor group outside the monopolised industry, if it takes out any part of its earnings in the monopolised product, must receive a lower rate of real pay per unit than it was receiving before. If the monopoly is monopoly by combination, all the centres inside the monopolised industry being of optimum size whether monopoly rules or not, the factor group there also, after monopoly has been introduced, will receive the same rate of pay as before in terms of the monopolised product. Hence, if it takes out any of its earnings in other products, it will receive a higher rate of real pay per unit than it was receiving before. If the industry to be monopolised is a single-centre industry, the factor group inside it may—if the centre under consideration is of less than optimum size it will—receive less pay per unit after monopolisation than before in terms of its own product, but it will certainly receive more pay per unit in terms of general value.

§ 7. So far we have treated the several factors of production in the lump. Let us next, still supposing that these factors are always and everywhere bound together into uniform factor-group units, consider them separately. The proportionate shares of the units that are outside the monopolistic industry are determined in accordance with the values of their marginal private products there. In the conditions we are supposing no change in this matter follows from the introduction of monopoly. But in the monopolised industry itself the factors, *qua* factors, while still being paid the value of the marginal private product that they would have elsewhere, are paid less than the value of the marginal

private product that they do have in the monopolised industry, and there is accumulated out of the balance a (normally substantial) net monopoly revenue. The contents of this must go to somebody. If they are distributed inside the monopolised industry in proportion to what the factors there earn without them, the relative earnings of these factors there are, of course, not modified. But, if they are absorbed in a larger proportion than this by some one factor, that factor makes a relative gain; and conversely. Thus, should managers annex the bulk of this monopoly revenue for their private use, the manager factor is advantaged: should they be acting as agents for capitalists and be forced to disgorge to them, capital is advantaged: should they be agents of labour, labour is advantaged. On the assumption here made, that the maintenance functions of the several factors are absolutely inelastic, so that their earnings do not affect their quantity, this is the end of the story. It should be noted, however, that, if that assumption were removed, there would be another important chapter dealing with the reactions of different distributions upon the stocks of the several factors.

§ 8. The question what happens to the *absolute* shares of aggregate real income, or, if we will, the absolute earnings received by the individual factors, whether inside or outside or both inside and outside the monopolised industry, is intractable in the same way as the corresponding question discussed in Chapter XXXVIII. §§ 5-6. It cannot be answered for any factor until we know in what particular commodity the owners of that factor elect to take out their earnings. Thus, suppose that, the relative shares (in money or anything else) enjoyed by two factors being unchanged, the owners of one of them consume exclusively the monopolised commodity and the owners of the other non-monopolised commodities. Since the monopolised commodity has, in consequence of the monopoly, been raised in value relatively to the others, the second set of owners may be receiving a larger absolute amount of what they buy than they used to receive, and the owners of the first set a smaller absolute amount of what they used to buy. This difficulty is quite general, and does not depend on either set

of owners purchasing exclusively monopolised commodities or non-monopolised commodities.

§ 9. Let us now remove the assumption of § 1, that the several factors of production are always and everywhere bound together in rigid uniform factor-group units. No general statement can be made about the way in which the effects of monopoly upon aggregate real income will be modified. But about the consequences to its distribution nescience is a little less complete. Whether or not the proportions in which the several factors engaged in the industry that becomes monopolised are altered in consequence of the restriction there, this restriction is bound to cause the proportion of those factors of which the monopolist industry employs an unusually large proportion to be increased in other industries, and that of those factors of which it employs an unusually small proportion to be diminished. The rates of pay of factors in any industry other than the monopolised industry, in terms of the product of their own industry, are not now left unchanged. If there were only two industries altogether and only two factors, the factor that extrusion from the monopolised industry rendered relatively redundant in the other would get a lower rate of pay in terms of its product there; the other factor a higher rate of pay. Unless, therefore, this result were upset by the former factor securing an abnormally large share of the net monopoly revenue acquired in the monopolised industry, it would obtain a smaller, and the other a larger proportionate share of the aggregate value of real income, whatever measure of value we use.[1] With more than two factors and more than two industries no categorical statements are admissible. Most individual factors will be rendered relatively redundant as against one lot of factors and relatively scarce as against another lot. In the absence of detailed knowledge the fate of particular factors is uncertain: much as we found it to be in § 8 of Chapter XXXIV. All that can be said in general is

[1] The shift in the relative values of the two factors will, of course, have a secondary effect in increasing the use in the monopolised industry of the one that has been relatively cheapened. But this only mitigates, it cannot reverse, the primary effect. Moreover, unless the monopolised industry is large, the mitigating effect is, in general, very small.

that, other things being equal, factors of which the monopolised industry employs an exceptionally large proportion are *prima facie* more likely to find their relative share of total value damaged by monopolisation than factors of which it employs an exceptionally small proportion. Factors whose relative share is damaged *probably* also suffer a reduction in their absolute earnings in terms of the things on which they expend their pay; but an adaptation of the argument of Chapter XXXVIII. shows that about absolute shares little can be known with certainty.

CHAPTER XLI

ALL-ROUND MONOPOLY SUBJECT TO ALL CENTRES OF PRODUCTION BEING OF OPTIMUM SIZE [1]

§ 1. On the line of argument followed in Chapter XVII. it is easily shown that the economic system is indeterminate if there is more than one monopolist engaged on a single commodity or on two commodities that are in any marked degree substitutes for or complements of one another. But the existence of one monopolist centre in each of several, or indeed of all, industries that make different commodities —more strictly, groups of commodities unconnected on the side of demand—was shown in Chapter XIX to be compatible with determinateness. It is, therefore, legitimate, and may prove instructive, to inquire what principal consequences would follow if each commodity, or, more strictly, each group of interrelated commodities, was produced under the aegis of a single monopolistic combination, instead of being produced under conditions of competition. We suppose, on the general lines of §§ 2-4 of the preceding chapter, (i) that the hours of work are not different under monopoly from what they are under competition, and (ii) that conditions are such as to preclude any factors from being attached anywhere in idleness. Our problem then has two aspects. The first relates to the comparative distribution, as between all-round monopoly and all-round competition, of real income as a whole among the several factors of production; the second to the distribution of factors of production among the industries engaged on different

[1] The subject matter of this chapter is discussed from a somewhat different angle in Mrs. Robinson's *Economics of Imperfect Competition*, chap. 27.

classes of commodities. These two aspects are interwoven; but it is possible up to a certain stage, with the help of simplifying assumptions, to investigate them separately. It is postulated that, in any event, the number of centres in each industry is so large that they are all of optimum size.

§ 2. Let us first envisage a many-commodity community that is completely symmetrical, in such wise that technical facilities for production and people's desire attitudes towards output are similar in all industries. In these conditions, alike under all-round competition and under all-round monopoly, the factors of production will be distributed evenly among the industries. The same amount of them will be engaged in each industry under the one system as under the other. Further, the demand functions in terms of general value, which are, of course, alike for all commodities, will be the same under both systems. This entails that the rates of exchange between the several commodities are the same under both systems. In these conditions it may seem at first sight that it can make no difference in any respect which of the two systems prevails. This inference, it is not difficult to show, is incorrect.

§ 3. Under all-round competition the hirer-sellers in each industry will push production to the point at which total sales receipts and total expenses of hiring the factors (including themselves at the current rate for persons of their quality) are equal. Under all-round monopoly they will push it to the point at which the excess of total sales receipts over total expenses of hiring the factors are a maximum. Now, except in the limiting case where the demand for the product, over the relevant range, has an infinite elasticity, it is impossible for this excess to be nil; it *must* have a positive value. But, as we have already seen, the quantity of factors engaged on each commodity is the same under all-round monopoly as it is under all-round competition. The only way, therefore, in which it is possible for the excess of sales receipts over total expenses of production to have a positive value is for the prices of the factors under all-round monopoly to be less than they would have been under all-round competition. That is to say, under all-round monopoly, in spite of the fact that

every centre is still of optimum size—nothing has happened to interfere with that—the factors, *qua* factors, are forced to accept rates of pay less than the value of their marginal private products, and the hirer-sellers, instead of receiving, *qua* hirer-sellers, nil remuneration, acquire for themselves what the factors lose. In other words, the factor that occupies the hirer-seller's seat is able, under all-round monopoly, to "exploit" the other factors. The extent of the exploitation is, as is shown in Appendix XIII. § 1, larger or smaller according as the elasticity of demand for product in respect of the quantity that is being produced—in our symmetrical system this elasticity is, of course, the same for all commodities —is small or large. If the elasticity is numerically equal to or less than unity, exploitation is complete, in the sense that the factors, apart from hirer-sellers, get no pay at all!

§ 4. To readers who like to picture to themslves a sequence of concrete happenings, this argument may be difficult to grasp. Anyone who finds it so should consider the process that will be gone through if, in a symmetrical system of the type we are imagining, which has so far acted on the rule of competition, a transition to all-round monopoly takes place. In each industry the hirer-sellers will cut down output, and so the quantity of collaborating factors that are employed, in such wise as to maximise net monopoly revenue. In all industries, therefore, large quantities of the collaborating factors are thrown out of work. But *ex hypothesi*, when adjustment to the new conditions has been made, they are all in work. This means that they are accepting rates of pay reduced in sufficient degree to make it worth while for hirer-sellers in the new conditions to engage them. Hence the persons who act as hirer-sellers are everywhere paid more, and the other factors collectively less, per unit than the value of their respective marginal private products. In every industry a larger share of that industry's output, and so of real income as a whole, goes to hirer-sellers, and a smaller share to the other factors. This is the first instance result. How far it is modified in the second instance by a redistribution of the net monopoly revenue by hirer-sellers among particular factors for whom they are agents depends, of course, on the detailed

arrangements on this matter that are made in each several industry.

§ 5. Let us now turn to the other aspect of our problem. It is obvious that in a completely symmetrical system, such as we have so far been supposing, the distribution of factors of production among industries will be precisely the same under all-round monopoly as under all-round competition. But in a system that is not completely symmetrical this is not obvious: indeed it is not true. In order to determine what is true, it is convenient in the first instance once more to suppose that there is only one factor of production, or, alternatively, that there are several factors bound together in rigid uniform factor-group units. Even so, it is plainly not possible to compare the distributions of these units among industries under all-round competition and all-round monopoly respectively, unless the principles by which the distributions are regulated are given. Under all-round competition, in a community in which there are no costs of transport either for factors of production or for commodities, distribution is governed by two rules; (i) that the rate of pay of factor-group units is the same in all industries, and (ii) that this rate of pay is equal in each industry to the value of the marginal private product, which, the centres being of optimum size, is the same as the average product, of the factor-group units assembled there. In the last chapter it was argued that, in order to make monopolistic action by combination in any one industry worth while, it must be open to the combination to prevent outsiders from starting new centres in that industry. All-round monopoly, however, is clearly possible even though no industry is free to shut its doors to outsiders. We are entitled to assume, and I hereby do assume, that under all-round monopoly, no less than under all-round competition, all industries are completely permeable by factor-group units. This entails that under all-round monopoly, just as under all-round competition, the rates of pay of factor-group units, in terms of general value, are everywhere the same. There is also a second rule, to be set against the rule that under all-round competition the rate of pay in every industry is equal to the value of marginal private

product there. Since each of our monopolies covers all the factor-group units engaged in the industry in which it stands, this second rule clearly is that the rate of pay in any industry is equal to the marginal collective product of value there; *i.e.* to the difference made by an extra unit to the value of all the factor-group units assembled in the industry. It is in the light of these rules that we have to determine in what, if any, respects the distribution of factor-group units among industries differs as between a régime of all-round monopoly and one of all-round competition.

§ 6. The working out of this problem entails a small manipulation of symbols. In respect of any industry R let us write η_r for the elasticity of demand in respect of the output produced under conditions of all-round competition, and E_r for the elasticity of average productivity in respect of that output. It is evident to common sense that the magnitude of the elasticity E_r must be expressible in terms of the relation between average (which is here the same as marginal private) product and marginal collective product. What the precise formula is it is, I think, beyond the powers of common sense to determine. But this is easily ascertained by algebraic analysis. In Appendix XIII. § 2 it is shown that, if we write, in respect of any output, P for the marginal collective product of an industry and p for the average product,

$$\frac{P}{p} = \left(1 + \frac{1}{E_r}\right).$$

Now in Appendix IV. it has already been proved that marginal product of value—whether private or collective—is equal to value of marginal product multiplied by $(1 + 1/\eta_r)$. Hence in industry R, for the output proper to all-round monopoly, marginal collective product of value is equal to value of average product multiplied by $(1 + 1/\eta_r)(1 + 1/E_r)$. In like manner in industry Q marginal collective product of value is equal to value of average product multiplied by $(1 + 1/\eta_q)(1 + 1/E_q)$.

§ 7. This step taken, there is no further difficulty. In conditions of all-round competition we have seen that values of average product are everywhere equal, and in conditions of

all-round monopoly that marginal collective products of value are everywhere equal. Let us imagine ourselves to start with a state of all-round competition, and in the first instance suppose that the only industries in existence are R and Q. In the initial state, if

$$\left(1+\frac{1}{\eta_r}\right)\left(1+\frac{1}{E_r}\right)=\left(1+\frac{1}{\eta_q}\right)\left(1+\frac{1}{E_r}\right),$$

not only the values of average product, but also the marginal collective products of value in the two industries are equal. Therefore, when transition is made to all-round monopoly, there is no need for any change in the quantity of factor-group units engaged in the two industries. Everything remains exactly the same as it was before. But, if

$$\left(1+\frac{1}{\eta_r}\right)\left(1+\frac{1}{E_r}\right)>\left(1+\frac{1}{\eta_q}\right)\left(1+\frac{1}{E_q}\right),$$

in order to satisfy the new condition, the factor-group units must be redistributed in such wise as to make the value of average product smaller in R and larger in Q. That is to say, the quantity of factor-group units engaged in R must be diminished, and that engaged in Q increased. This result is readily generalised. When there are a number of different industries, more factor-group units will be engaged under all-round monopoly than under all-round competition in industries where, in respect of the output proper to all-round competition, $(1+1/\eta)(1+1/E)$ is abnormally large, and less in those where it is abnormally small. There will be a certain critical intermediate value, which we may call V, such that in all industries, for which under all-round competition $(1+1/\eta)(1+1/E)>V$, more factor-group units will be assembled under all-round monopoly than under all-round competition; and in all industries of opposite type less factor-group units will be assembled. What the precise value of V is depends, of course, on the detailed circumstances. We have not said, it will be noticed, that, among the industries that expand, those in which, in conditions of all-round competition, $(1+1/\eta)\,(1+1/E)$ is larger, will expand more than those in which it is smaller. This inference, and the corresponding inference about industries that contract, is only legitimate

provided that the relevant curves are constant elasticity curves. On our data we only know the values of the η's and the E's in respect of the particular output proper to all-round competition.[1]

§ 8. Now elasticity of demand is always negative. Hence the value of $(1 + 1/\eta)(1 + 1/E)$ will be larger in any industry, the more elastic is the demand for its product. The elasticity of marginal collective product is infinite if, the centres of production being, as we are here assuming them to be, of optimum size, no external or external-internal economies or diseconomies are available. It is positive if external or external-internal economies are available. But, since external and external-internal diseconomies do not in practice occur, it cannot be negative. Hence the value of $(1 + 1/\eta)(1 + 1/E)$ will also be larger in any industry, the less elastic is the marginal collective productivity, *i.e.*, on the present hypotheses, the more steeply increasing return operates. Now the relation between elasticity of demand and elasticity of marginal collective productivity is almost certainly random. That is to say, there is no reason to suppose that high elasticity of demand is associated with large external or external-internal economies, or *vice versa*. We may, therefore, conclude that all-round monopolisation is likely to make the quantity of factor-group units engaged in industries of highly elastic demand, and also that engaged in industries of strongly increasing returns, larger than they would have been under all-round competition; and conversely. All-round competition and all-round monopoly would only be associated with the same distribution of factor-group units if the elasticities of demand and the elasticities of marginal collective productivity under conditions of all-round competition were identical in all industries, or if, differences existing, the differences in the one kind of elasticity exactly cancelled out those in the other kind. Such situations could obviously only exist by a miracle.

[1] The same results can be expressed otherwise. It is proved in Appendix XI. § 3 that, if e be written for the elasticity of supply in the sense of average costs, $\frac{1}{1 + 1/e} = (1 + 1/E)$. Therefore, if we so wish, we are free to substitute the former for the latter expression throughout the argument of the text.

§ 9. So far we have assumed that the several factors of production are everywhere bound together in the same rigid uniform factor-group units. If this assumption is abandoned and it is recognised that the factors are combined in different proportions in different industries, the broad results obtained above are left intact. Now, however, there will be further happenings. The factors, which are predominantly employed in industries that expand when all-round monopoly is substituted for all-round competition, will come to be paid more than before relatively to the others in terms of general value. As an indirect consequence of this, these industries will expand less than they would have done otherwise. Hence the shift caused in the distribution of resources among industries, and so in the relative outputs of different industries, by the substitution of all-round monopoly for all-round competition is likely to be smaller in actual life than in our hypothetical world of rigid uniform factor-group units.

§ 10. The observation just made, that the prices of the factors predominantly engaged in industries which all-round monopoly expands will be raised relatively to the prices of other factors, brings us back into contact with the first aspect of our problem. In § 3 it was shown that in a completely symmetrical system the substitution of all-round monopoly for all-round competition will entail the general body of factors being exploited in the interests of the particular factor to which the hirer-sellers are attached. Obviously this is still true in a system that is not symmetrical. But in such a system, we have now also learned, the factors which are damaged will not, in general, be damaged in equal measure, but, according to the nature of the shift in distribution among industries, some of them will suffer more than others.

CHAPTER XLII

MONOPOLY IN THE INTEREST OF CONSUMERS SUBJECT TO CENTRES OF PRODUCTION BEING OF OPTIMUM SIZE

§ 1. THE persons in control of a municipal tramway or waterworks are often in a position to exercise monopoly power. But, being agents of the consumers, they presumably act, not in their own interest or in that of the factors of production that collaborate with them, but in the interest of the consumers of their products. The consequences of monopoly directed to this end will clearly be different from those of ordinary producers' monopoly as studied in the two preceding chapters. A brief comment upon them is needed to round off the analysis of those chapters.

§ 2. Maintaining the assumption of a perfect market with nil cost of transport, we rule out price discriminations as between rich and poor purchasers, or between purchasers with keener and less keen desire for the product. More generally, we ignore differences of wealth and temperament among the public. With that proviso the public monopolist, if he concentrates attention on his own industry, will aim at maximising the money value of his purchasers' total consumers' surplus minus the money cost of production. To do this he must produce that quantity of output which will make demand price equal to marginal collective supply price; that is to say, appropriate units being chosen, to the price of the increment of factor-group units required to produce the marginal collective product of the industry. If this marginal collective product is equal to the average product, the output is the same as would have been produced, all other things being equal, under the rule of competition. If the marginal

collective product is larger than the average product, the amount produced will be more than this; in the converse case less.

§ 3. This result is complete and self-contained. But, nevertheless, a caution against misinterpretation may perhaps not be out of place. Provided that variations in output leave the relative values of the several factors and the proportions in which they are engaged by our public monopolist unaltered, excess of marginal collective over average product, equality with it, and deficiency below it respectively imply, and are implied by, increasing, constant and diminishing returns. Therefore we may say that the public monopolist will produce more, the same amount as, or less than, would be produced under competitive conditions according as increasing, constant or diminishing returns prevail. But, if the above proviso is not satisfied, we must not say this. For, as was pointed out in Chapter XXVII. § 10, departures from constant returns may then be a consequence of variability in the prices of the several factors and of the proportions in which they are engaged in the industry affected: and these departures have no relevance to our problem.

§ 4. It remains to observe that a public monopolist *may* adopt a wider outlook than was supposed in § 2. Many industries throw costs or confer benefits in an indirect way upon persons other than those who enter into contractual relations with them. Hence, alongside of marginal collective product, there stands marginal *social* product, in which these incidents also are reckoned. A public monopolist, if he looks beyond his own industry to economic welfare as a whole, will so arrange his output as to make demand price equal to marginal *social* supply price. According, then, as the marginal social product of his concern is greater or less than the marginal collective product, he will produce a larger or a smaller output than the narrower-visioned public monopolist of § 2. The main interest of these results is not, however, for our present inquiry, but for the economics of welfare.

CHAPTER XLIII

COST OF TRANSPORT FOR COMMODITIES AND PRICE DISCRIMINATION BY A MONOPOLISTIC INDUSTRY

§ 1. Up to this point in our study of a many-commodity community we have adopted the highly artificial assumption that there are no costs of transport either for commodities or for factors of production other than land. In this and the two following chapters we have to investigate certain consequences that may follow for particular industries, in which the rule of monopoly is followed, when, factors of production still being supposed to move freely, the existence of costs of transport for the commodities in which the industries deal is no longer denied. In the present chapter the consequence that concerns us is price discrimination.

§ 2. On the definition of a market given in Chapter XV. it is impossible for any commodity to have, at the nodal point of any market, more than one price. Moreover, the different parts of the output of any seller cannot have two prices "at works", even though they are sold in different markets, so long as the cost of transport between the nodal points of the two markets does not exceed the difference between the costs of transport from his works to the two nodal points. If there are no costs of transport at all, this condition obviously must be satisfied. But if there are costs of transport between the two nodal points, or, to put the matter in other terms, if one buyer cannot act as a middleman for another buyer without thereby incurring transport charges, the condition need not be satisfied. If the direct costs of transport from a seller's works to each nodal point separately are nil, and yet costs of transport from the one nodal point

to the other are not nil, it *cannot* be satisfied. It is then open to the seller to charge different prices at works for sales to different nodal points without his policy being defeated by the penalised buyers refusing to buy from him direct and making all their purchases via the other.[1] In actual life there are a great many situations of this kind. A notorious example is afforded by those kartels, which, defended against the re-import of their goods by freight charges and tariffs, sell abroad at lower prices than at home.[2]

§ 3. When the markets between which discrimination is to be practised have been demarcated—the demarcation, though to some extent within the monopolist's own discretion, is in part decided for him by the technical situation —the prices that it will pay him to charge in the separate markets are related to one another in a simple manner. If we write $p_1, p_2 \ldots$ for the prices and $\eta_1, \eta_2 \ldots$ (which are, of course, negative) for the elasticities of demand, in respect of those prices in the several markets, an easy extension of the argument of Appendix XII. § 2 yields the formula

$$p_1\left(1+\frac{1}{\eta_1}\right)=p_2\left(1+\frac{1}{\eta_2}\right)= \ldots => 0.$$

That is to say, higher prices will be associated with (numerically) smaller, and lower prices with (numerically) larger, elasticities.[3] If we know that the demand curves of the several markets are all constant elasticity curves, this formula enables us to arrange the prices of the several markets in a definite order. But otherwise, since in the same market different prices are associated with different elasticities, it does not enable us to do this. It is thus more elegant than informative. To learn anything of importance we must be given the relevant functions in full.

§ 4. The relation between the aggregate quantities of out-

[1] Cf. *ante*, Chapter XV. § 11.

[2] Cf. for a fairly full account of this matter, *The Economics of Welfare*, Part II., chap. xvii.

[3] Cf. J. Robinson, *The Economics of Imperfect Competition*, p. 187. It will be observed that the condition, $p_1\left(1+\frac{1}{\eta_1}\right)$ and so on $= > 0$, precludes any of the η's from having a value numerically < 1.

put that will be produced by a monopolist constrained to charge one price only and by one who is free to discriminate between markets varies in different circumstances. First, let the conditions be such that under single-price monopoly some of the commodity would be purchased both in market A and in market B—these two being supposed for simplicity to be the only markets available. There are then no adequate grounds for expecting either that output under price discrimination will exceed, or that it will fall short of, output under single-price monopoly. If the curves of demand and supply are straight lines, the two outputs will be equal. Secondly, let the conditions be such that under single-price monopoly some of the commodity would have been purchased in A, but none in B. It is then impossible that the introduction of discriminating power should lead to diminished output. On the contrary, if there is any substantial demand in B, it must lead to increased output. The amount of the increase will be especially great if the demand in B is elastic and if the commodity is produced under the rule of increasing returns. Finally, let the conditions be such that with single-price monopoly none of the commodity would have been purchased in either A or B. It is again impossible that the introduction of discriminating power should lead to diminished output. It *may* lead to increased output. The condition for this is the same as that—to be mentioned in the next paragraph—which enables discriminating monopoly to provide some output though under competition there would be none.

§ 5. When there are no economies of large scale it is obviously impossible for discriminating monopoly to make output greater than it would be under competition: it *must* make it smaller. When, however, economies of large scale are available, and, in consequence, the commodity is produced under conditions of increasing returns, the question is more complex. If the markets are demarcated in such a way that the monopolist can and does charge different prices for every single unit that he sells, output under discriminating monopoly must be larger than under competition. With markets less completely atomised, output may be either

larger or smaller than under competition. It is more likely to be larger, the more nearly the number of markets into which demands are divided approximates towards the number of units for which any demand exists. In conditions where monopoly *plus* discrimination would yield a return larger than would be obtained under competition there may exist some output for which the return under monopoly covers costs of production, but no output for which the return under competition does this. When that is so, discriminating monopoly will provide some output, though competition would provide none. In view, however, of the limitation, which practical considerations impose, alike upon the number of markets that can be formed and upon the monopolist's freedom to make up the several markets in the way most advantageous to himself, this will very rarely happen. In general, it is exceedingly improbable that in an industry selected at random monopoly *plus* discrimination will provide an output as large as would be provided under competition.[1]

[1] The two last sections are taken in the main from Part II. chap. xvii. §§ 13-14 of *The Economics of Welfare*.

CHAPTER XLIV

COSTS OF TRANSPORT OF COMMODITIES AND MULTIPLE MONOPOLY INSIDE AN INDUSTRY

§ 1. We have now to suppose that in one industry there are a number of centres of production, but that costs of transport prevent there being a single market for all of them. For simplicity let us imagine that each centre has a separate market, *i.e.* that the nodal point of each market is the works of the single centre that serves it. It is postulated that factors of production other than land can move quite freely from any one place or industry to any other.

§ 2. If in our industry the factor-group units in the centres established there, acting, as, of course, they will, on the rule of monopoly, obtain higher rates of pay than are available in other industries, new centres will be tempted to establish themselves in places intermediate between those occupied by existing centres, so as to capture a part of the markets that have so far been attached to these. With factors of production other than land completely free to move this is inevitable. The process will tend to be carried exactly to the point at which each factor (including the hirer-sellers) in every centre is remunerated at a rate equal to what similar units receive elsewhere. This tendency may not, of course, be completely realised. For, whereas with a system of n centres, the factors might be getting rather better pay than is available in other industries, with a system of $(n+1)$ centres they might be getting rather worse. When we imagine two centres *already established* and the question arises whether or not a new one shall be planted down between them, this rough-edge seems formidable. But from the standpoint of a station-

ary state the problem should not be looked at in this way. We must rather imagine ourselves starting with a clear board. The tendency we have to consider is the tendency for centres to be established in such a number and so distributed as to make the rates of pay to the factors engaged in each of them equal to what is obtainable elsewhere. If the aggregate quantity of product required is large relatively to the optimum size of a centre, and *a fortiori*, therefore, to any size less than the optimum, the rough-edge is of trifling importance. In general, we may safely leave it out of account and, for practical purposes, reckon, in the manner of Chapter XXIV. §10, that the tendency *is* completely realised.

§ 3. In the situation just described it is plain that the markets of the different centres are not all isolated from one another. On the contrary, each centre, if we suppose the purchasers of our commodity to be distributed symmetrically over the country, will serve a more or less circular area, whose boundaries touch those of the markets of several other centres. That is to say, every market is connected with several other markets. If, therefore, as in these circumstances we may expect will happen, each centre acts on the rule of monopoly, there is set up a system of the type contemplated in Chapter XVIII. Systems of that type, as we have argued, may or may not be determinate. With a fair number of centres spread over a considerable spatial range, the analysis of Chapter XVIII. §§ 5-6 showed that it is quite likely to be determinate. In the discussion that follows I assume that the conditions necessary for this are in fact satisfied.

§ 4. Before we proceed further a difficulty must be cleared out of the way. In Chapter XXXVI. § 6 it was shown that, when a centre of production acts on the rule of monopoly, it will so regulate output as to maximise aggregate sales receipts *minus* aggregate costs—cost in respect of each factor being measured by its quantity multiplied by what it can earn elsewhere. In § 3 above, we found that in every centre the factor-group units must earn precisely what they earn elsewhere. Is there not an inconsistency here? Have we not, in effect, more independent equations than there are unknowns to be determined? The solution of this difficulty is

simple. The number, and so the sizes, of the centres of production are not fixed, but are free to adjust themselves to the surrounding conditions. The double equality stipulated for is brought about by the number of the centres automatically becoming such that the rates of remuneration which result from maximizing net monopoly revenue coincide with the rates which are being paid for the factors in other industries. There is, in short, in the background, alongside of the number of factor-group units engaged in the industry a second unknown—the number of centres. This makes unknowns and independent equations match each other, as they ought to do.[1]

§ 5. The fact that in our industry the factors in each centre obtain the same rates of pay as rule in other industries implies, that industry being supposed small relatively to the sum of all industries, that they obtain the same rates as they would have obtained under competition. Since, then, a centre acting monopolistically sells at a higher price than it would charge if there were no monopoly, it follows that under monopoly the average product of the factor-group units in each centre must be smaller than it would be under competition. Hence, granted that under competition each centre would be of optimum size, under monopoly of the type we are here studying each must be of less or of more than optimum size. It cannot be of more than optimum size; for, if it were, more centres would be formed. Hence it must be of less than optimum size. That is to say, under generalised monopoly by the separate centres of an industry that is split into many markets by costs of commodity transport, the number of centres will be larger than it would be under all-round competition; the marginal private product of the factor-group in each centre will be smaller than, instead of equal to, the average product; and, as this last statement implies, for a given volume of factors of production engaged in an industry, aggregate output will fall short of the maximum of which physical conditions would permit. It follows, as a corollary, that, whereas in a

[1] Cf. Appendix XIV. § 1. When attention is concentrated on one industry, this is sufficient. In order to secure complete determinateness for the whole system of industries, we must add to the assumptions set out above the further assumption that in at least one industry conditions are such as to make the centres of production there of optimum size.

perfectly symmetrical system, all-round monopoly, as described in Chapter XLI., *may* leave total output the same as it would have been under all-round competition, monopoly of the type here envisaged, extended to all industries, by reducing the scale of the typical productive centre below the optimum, necessarily makes total output less than it would have been.

§ 6. In the preceding sections we have been considering factor-group units as wholes without regard for their component parts. In the main we may rest content with this. But there is one point to which attention should be called. When, under the influence of monopolistic action, a centre is reduced below optimum size, this entails, not merely that average cost would be reduced if the quantities of all the factors were increased in a common proportion, but also that it would be reduced if, the quantities of the other factors being what they are, the quantity of any one factor were increased. That is to say, equipment being given, the stock of labour is less than the stock which, with that equipment, would reduce average cost to a minimum. In like manner, the stock of labour being given, the stock of equipment is less than the stock which, with that stock of labour, would reduce average cost to a minimum. This is the generalised form of a proposition, on which Mr. Harrod and others have laid stress, to wit that, under the conditions of what they call imperfect competition, not only will the lay-out of firms be less than the optimum, but also this lay-out will be employed to less than its optimum capacity.[1] An algebraic proof is given in Appendix XIV. § 1.

§ 7. It is important to determine how large a difference to the output of a given collection of factors in any industry the supersession of the rule of competition by generalised monopoly, in the manner here described, might be expected to make. For to know this is to know how close an approximation to the truth will be obtained by treating an industry, in which generalised monopoly in fact operates, *as if* it were subject to the rule of competition. The answer is that in a symmetrically situated industry, *i.e.* one in which all centres

[1] Cf. Harrod, "Doctrines of Imperfect Competition", *Quarterly Journal of Economics*, May 1934, p. 451.

of production are of equal size, monopolistic policy cannot raise average cost of production in terms of factors above what it would be under competition by more than the cost of transport of the commodity between adjacent centres.[1] For, if in any centre average cost were raised more than this, the adjacent centre, by assuming a larger scale, would be able to undersell it in its own market and make a profit in doing so. Thus, if the cost of transporting the commodity between adjacent centres is 5 per cent of the cost of producing it in either of them, this kind of monopolistic policy cannot reduce the productivity of a given quantity of factors engaged in the industry by more than 5 per cent of what it would have been under competition. Plainly, therefore, the error, that results from postulating the rule of competition in an industry which is in fact governed by multiple monopoly of the kind here described, varies inversely in magnitude with the general level of transport costs. The better and cheaper the means of communication, the slighter the error is likely to be.

§ 8. So far in this chapter we have tacitly assumed that the markets open to different centres of production in an industry can only be separated if and in so far as costs are entailed by the physical transport from place to place of the commodity in which the industry deals. In earlier chapters, however, it has been pointed out that preferences on the part of particular buyers for trading with particular sellers, whether these are based on solid grounds or on mere custom-engendered prejudice, act in much the same way as costs of transport. There can be no doubt that for short-period problems these preferences play a very important part. For the long-period analysis proper to a study of stationary states the main part, though not perhaps the whole of them, should, I think, be regarded as smoothed away. If, however, they are not so regarded, there is no difficulty in bringing them into account on the same footing as costs of transport.

[1] Of course, the average cost in a centre may be raised by *less* than this. That will happen if the cost of transport between two centres is greater than the excess of the price proper to monopoly over that proper to competition. In this case, however, the markets are no longer connected, and we have not the type of situation we have been studying in this chapter, but separate single monopolies.

CHAPTER XLV

A MANY-COMMODITY COMMUNITY AND LOCATION OF PRODUCTION

§ 1. Costs of transport for commodities have now to be brought into account from a much wider point of view. In Chapter XXXV. something was said about the way in which industry will tend to locate itself in a one-commodity community (i) when factors of production can and (ii) when they cannot move freely. For the purpose of that analysis there could, of course, be no question of moving a product made in one place to exchange it against products made in other places. For a many-commodity community the problem is much more complicated. The ease or difficulty of moving commodities between places comes into the picture and plays a dominant part. In this chapter I shall attempt a very brief outline sketch, taking as premise the postulate that, while land is, of course, fixed, other factors of production can, from the standpoint of a stationary state, be located in the optimum manner without regard for transport costs.

§ 2. One preliminary observation of a general kind must be made. When the transport of commodities does not entail costs, the proposition that each factor of production must be so distributed that its marginal private products are equal everywhere, though not in all circumstances true, is in all circumstances unambiguous. When the transport of commodities does entail costs, this is no longer so. If, indeed, there is a single point, to which all commodities have to be delivered before they are passed to the people who finally consume them, and at which their values are related in some specific way, there is no difficulty. Each factor must be so

distributed among places and commodities that the marginal private products of all of them have in the central market, after transport charges have been paid, the same values, whether in terms of money or of anything else. But, if, instead of one single point, through which all units of all commodities pass, there are a number of centres of distribution, in which the relative values of two or more commodities are different, no unambiguous test for equality between the values of the marginal private products of a factor in two different places and engaged on different commodities can be found.

§ 3. With this preliminary, let us proceed to our outline sketch. To attempt this at the level of abstraction which we have adopted hitherto, with finished commodities made complete, so to speak, out of the air in single centres, would be a waste of time. We must descend to the real world and reckon with the fact that actual finished commodities are made out of, and with the help of, raw materials and natural products, which are found, some in one place, some in another, and that often, in the course of manufacture, the growing embryo of a finished product passes through, and is worked at, in several different centres. The problem of location in this setting was studied in a very illuminating way by Alfred Weber as long ago as 1909, but English economists have devoted little attention to it. The following section indicates very briefly the general line of Weber's approach.[1]

§ 4. Let us concentrate attention on a single commodity selling in a single market, and let us postulate that the location and the size of this market, and also the regions in which the materials used in making it can be obtained, are given. Weber distinguishes between materials that are "ubiquities", *i.e.* obtainable everywhere, and those that are localised in particular regions. Within each group he distinguishes further between "pure materials", which are embodied whole in the finished commodity, and weight-losing materials, which operate on the commodity in some degree—in the extreme case of fuel *in toto*—without becoming embodied in it and constituting a part of its weight. For industries

[1] His work was translated into English with the title *Theory of Location of Industry* by Dr. Friedrich in 1929.

in which only one material is used he shows that, if this material is a ubiquity and if it is embodied in any degree into the finished product, producing centres will be set up in the regions of consumption, because, the nearer they are to consumers' homes, the less ton-miles of weight have to be transported and, therefore, in general, the less charges are incurred in delivering a given quantity of commodity at these homes. *Per contra*, an industry which makes use of a localised weight-losing material—coal or other fuel—will be set up in the region where the material is found, because, if it is established anywhere else, a cost, which might be dispensed with, is incurred in transporting the material. Finally, an industry which makes use of a pure material, provided that charges for transport are proportional to weight, may be situated indifferently at any point on the line between the region of consumption and the region where the material is found; because, whatever its position on this line, the same total ton mileage of transport has to be accomplished per unit of finished product delivered to the consumers. If, however, transport charges per ton are higher for more valuable than for less valuable products, cost can be saved by an industry of this kind establishing itself in the region of consumption; and it will establish itself there. Thus for industries that make use of only one material the problem of determining their regional location is a simple one. When several materials are used it is more complex. The employment of "ubiquities" as pure materials pulls the industry affected towards the region of consumption; the employment of any localised weight-losing material pulls it towards the region where that material is found; the employment of more than one localised pure material pushes it towards the consumption market, because, if it is established anywhere else—except in the special case where a straight line between the consumption point and the production point of one pure material passes through the production points of all the others—some of the pure material, whether in native form or partly worked up, has to make a roundabout journey. The interaction of these pulls and pushes determines the location of the industry. When the degree to which the various materials employed are weight-losing and

the plan on which transport charges are framed, together with the facts about economies of large scale, are given, the most economical and, therefore, on our assumptions, the actual regions in which the producing centres of our industry will settle are thereby determined.

§ 5. It is easy to see that an analysis of this kind can be extended, though with growing complication, to cover cases where there are a number of different consumption markets and a number of different places at which one or another of the materials used in an industry are found. Further, *provided that the consumption markets are given independently of the location of the several industries*—provided, that is to say, that the places where people live are determined wholly without regard to where they work—it can be extended to cover, not merely one industry, but the whole body of industries; though here a further modifying influence is present, in so far as advantage may be gained by industries that make use of like materials being assembled close to one another. It is, however, self-evident that this proviso cannot be satisfied. For everybody's residence is determined in some measure by his place of work, and, for most people, it must be within a very short distance of that place. Hence, while the position of the consumption markets helps to determine the position of the industries, the position of the industries in turn helps to determine the position of the consumption markets. Indeed, broadly speaking, each industry is a consumption market, and there are no positions of consumption markets that are not positions of industries. Hence to assume, in studying the location of each industry, that the relevant consumption market is given is, in effect, to assume that the location of all the other industries is given. Here, as in all the most fundamental problems of economics, we have to do, not with a chain of causes, but with a system, all the parts of which mutually determine one another.

CHAPTER XLVI

THE CONSEQUENCES IN A MANY-COMMODITY COMMUNITY OF VARIOUS CONDITIONS OF TRANSPORT COSTS FOR COMMODITIES WHEN FACTORS OF PRODUCTION ARE IMPERFECTLY MOBILE

§ 1. We have still to consider, for a many-commodity community, what consequences, over and above those described in Chapter XXXV. for a one-commodity community, follow when the assumption that factors of production are able to move freely without costs is abandoned. Let us begin by supposing that, apart from land, they can all so move within a limited region, but are debarred absolutely from overpassing that region's boundary. This is the state of things contemplated in the pure theory of foreign trade in its simplest form. Obviously the implications of it will be different according as commodities can move freely or are themselves also subject to greater or less costs of transport.

§ 2. We have, then, to compare two situations in which commodity transport respectively costs something and costs nothing—more generally, in which it costs more and in which it costs less.[1] Plainly, if the lower set of costs, as well as the higher set, is too high to allow of trade, it is immaterial which of the two rules. Let us suppose, therefore, that the lower set allows some trade which the higher did not allow. What are the consequences? To obtain a broad view, let us imagine all products in the two regions to be divided into two groups,

[1] We are here confining attention, it will be understood, to genuine differences in the physical cost of moving commodities. Differences in the charges that people conducting movement have to meet in respect of these costs, on account, *e.g.*, of tariffs, bounties and so on, are not here in question. Some reference to these matters is made in Chapter XLVII.

represented respectively by wheat and iron; of which iron is a potential export from A to B and wheat a potential export from B to A. After the lower set of transport costs has been substituted for the higher the same total quantity of factors of production in A and B together has power to produce, and to deliver at their final destination, more of *both* of these two things than it had the power to do before. For, in effect, both A and B have become technically more efficient at production in both lines. But it does not follow that a larger total quantity of both wheat and iron must be produced. For suppose that tastes in A are such that practically the same amount of wheat is called for whatever the effort required to produce it, *i.e.* that the desire for wheat is highly inelastic, while in B a precisely contrary state of things prevails; and that the desire for iron is fairly elastic in both countries. Let the conditions of production in the two places be such that, prior to trade, wheat is much more valuable relatively to iron in A than it is in B; and also that, the larger the proportion of either commodity that is being produced in either place, the higher is its supply price in that place relatively to the supply price there of the other commodity. When trade is opened up, iron from A is traded against wheat from B; iron becomes more valuable in A relatively to wheat; and wheat becomes more valuable in B relatively to iron. In these conditions A will consume practically the same amount of wheat as before in spite of getting it cheaper, while B will consume substantially less wheat than before because to him it is dearer. Thus the total consumption—which is, of course, equal to the total production—of wheat will be less than before. On the assumption that the stocks of the factors maintained and at work in each region are unaltered, this, of course, implies that the total quantity of iron produced and consumed is larger than before. Thus the total output over the two regions together of at least one of our two representative commodities must be enhanced, but not necessarily the total output of both. Generalising this result, we conclude that, factors of production being completely immobile between two regions, with lower costs of transport for commodities the total output

of some commodities must be larger than it would be with higher costs, and the total output of all commodities may, but not must, be larger.

§ 3. About the relative shares of real income that accrue to the several factors of production more definite results can be reached. In so far as trading of commodities between different regions takes place, a region that exports any commodity produces more of it than it would otherwise do, and a region that imports any commodity—unless in any event it would produce none of it—produces less of it than it would otherwise do. If the proportional parts played by the several factors are the same everywhere for every commodity and in respect of every quantity of output, variations in the relative values of commodities can have no effect on the relative values of the marginal private products, *i.e.* on the relative rates of pay, of the several factors. The reason, however, why, in the absence of trade, the relative values of commodities as between A and B differ, frequently is that in the manufacture of one of them a factor, that is redundant and relatively cheap in A, plays a principal part, while in the other a different factor, abundant and relatively cheap in B, plays a principal part. In such circumstances a lowering of the costs of commodity transport, that allows interregional trade to develop, will entail an enhanced demand in each of the two regions for the factor which is there abundant and relatively cheap. Hence the relative values of these factors in A and B respectively will stand nearer together if trade is established than they would do in the absence of trade. The opening up of trade between the agricultural products of the new world and the manufactures of this country made the ratio of the annual rent per acre of agricultural land to the annual wage per representative workman much smaller here and somewhat larger there than it would have been if this country had continued in the main to produce its own food.

§ 4. Up to this point we have been supposing that, while movement of commodities between regions is partially impeded by costs, movement of factors of production is absolutely estopped. Let us now allow costs of transport to

hamper in some measure, but not to bar out absolutely, the movement of factors of production other than land. There emerges a new point of some practical importance. We have seen that, with factors immovable, the smaller are the costs of transport for commodities, the less the rates of pay for given quantities of factors, in terms of general value, will differ in different places. With factors movable, some movement of them, which it would have been worth while to undertake had the costs of moving commodities been high, will not be undertaken if these costs are low. Thus low costs of movement for commodities allow commodity movement to be substituted in some measure for movement of factors of production. For example, when low cost of commodity transport enables workpeople in old countries to obtain food cheaply, they are less likely to emigrate abroad than they would be otherwise. Consequently, after adjustment has been made to the lower costs, the stock of labour in old countries is likely to be larger relatively to the stock in new countries than it would have been had high costs of commodity movement continued.

§ 5. To the proposition, that easy commodity movement makes against factor movement, a sense of symmetry inclines one to add the companion proposition, that easy factor movement makes against commodity movement. But this step is not warranted. No doubt, if, as a consequence of low costs of factor movement, region A is left derelict of men and equipment, trade between A and elsewhere must languish. Cheap movement of factors may mean, however, not a general efflux of factors of production from A, but an efflux of some factors accompanied by an influx of others. Thus a country well supplied with capital and poorly supplied with labour may send some capital to other countries and bring some labour from them. In cases of this kind the presumption is that, when the transition has been consummated, commodity trade will be larger in the stationary state appropriate to the new conditions than it was in that appropriate to the old.

CHAPTER XLVII

PRODUCTION AND DISTRIBUTION IN RELATION TO STATE ACTION

§ 1. Though it is no part of the purpose of this book to study public finance, we can hardly end the discussion of our last and most complex model without a word on that matter. For the conduct of governments is bound to exercise an important influence upon the volume of real income and its distribution among the factors of production. True, since we are concerned only with stationary states, government loans do not come under review; but regular annually recurring State actions plainly do so come. So much money is raised every year by the State through various kinds of taxation, and is expended by it either in the provision of public services, such as defence, education and public health, or in making gratuitous transfers to certain categories of people, holders of war-loan stock, pensioners, unemployed persons (in respect of the Treasury contribution) and so on, or in giving bounties or subsidies to particular forms of productive activity. We have to indicate in a general way the relation in which these operations stand to our scheme of analysis.

§ 2. Their most obvious effect is to modify the distribution of productive resources among different sorts of commodities. First, a certain quantity of purchasing power is taken from the tax-payers by the State and devoted to public services. Instead, therefore, of the things that they would have bought for themselves, these tax-payers enjoy their share of the public services—defence, education, or whatever it may be. What they give up is probably in part something that, had the public service not existed, they would have

tried to provide for themselves in place of it. Thus, if there were no police, private persons would probably go about armed; while a public education service and a public health service are in some measure substitutes for attempts, which, in their absence, would be made to operate private ones. It is conceivable that the productive resources of the community will be engaged in making precisely the same things and rendering precisely the same services as they would be doing if the State were wholly quiescent. But this is exceedingly unlikely. In particular, the amount of these resources devoted to national defence is certain to be much larger under public than it would have been under private enterprise. Secondly, a certain quantity of purchasing power is taken in taxation from one set of people and transferred in war-loan interest, pensions and so on, to another set. It is conceivable that the transferees will use their purchasing power in the same way that those from whom it is taken would have used it had it been left in their hands. In this case all the factors of production are occupied exactly as they would have been, though the proceeds of their activity are enjoyed by different people. But this again is exceedingly unlikely. It is practically certain that the direction of demand will be modified, and different commodities, therefore, produced. Transfers, for example, from the payers of super-tax to old-age pensioners will entail an enhanced manufacture of necessaries and comforts as compared with luxury goods. Thirdly, a certain quantity of purchasing power is taken from the tax-payers and used in subsidising the production of particular commodities. A necessary consequence is that more resources are devoted to making these commodities and less to making others; and—the other side of the shield—that more of these commodities and less of others is purchased and consumed by the community as a whole.

§ 3. So far we have tacitly assumed that it is only through the use to which the money raised by the State is put that the distribution of productive resources among commodities is affected. In truth, however, this distribution is also affected by the manner in which the money is collected. Direct taxes, such as income tax and death duties, do not, in a stationary

state, penalise the purchase of any one sort of commodity as against any other; nor would an expenditure tax assessed at the same rate on all commodities, including direct services, do this. But taxes assessed on the purchase of particular commodities, beer, tobacco and so on, by making these commodities more expensive than they would otherwise have been, discourage people from consuming them, and so cause productive resources to be diverted in some measure from them to other commodities that are not taxed or are taxed less heavily.

§ 4. As we have had to recognise more than once in other connections, two real incomes, one of which contains more of some commodities and the other more of other commodities, can, in principle, be compared as contributors to economic welfare, but cannot be compared directly as physical entities. Hence the question whether any given system of State activities makes the real income of a community larger or smaller than it would have been in the absence of that system, like the corresponding question about monopolisation, must, with the definitions we are here using, be left unanswered.

§ 5. As we saw in Chapter XXXIX., any shifting of production from one collection of commodities to another is liable to alter the proportion in which the value of aggregate output is earned by the several factors, increasing that of those which play a relatively more important part under the new scheme. An enlarged proportion of the value of the aggregate output does not, however, necessarily imply an increased absolute quantity of the particular collection of things on which any given factor spends its money earnings. The State's action *may*, while keeping aggregate money income constant, increase a particular factor's proportion of the value of output from, say, 30 per cent to 33 per cent, *i.e.* by 10 per cent of its original amount, while, at the same time, increasing the price of the things which our factor buys by more than 10 per cent. Moreover, in so far as a State engages resources in the upkeep of public objects, such as battleships, a part of what the factors make is not allocated among them at all, and can only be brought under an analysis of distribution by means of some more or less

arbitrary convention. Finally, we must be on our guard in this connection against confusing earnings with receipts. In so far as the State redistributes income among various classes of people—*e.g.* taxes rich rentiers to provide pensions for the poor—the distribution of *receipts* is, of course, made different from the distribution of *earnings* acquired by people in their capacity as owners of so much of one or another factor of production.

§ 6. To carry the discussion further would entail abandoning the assumption, adopted as a device of exposition in this group of chapters, that the maintenance functions of the factors are perfectly inelastic. No account of the consequences of State action can be adequate unless that is done. For a main issue in discussions of public finance concerns the reactions of various types of State action upon the stocks of these factors. These reactions may be expected to differ according to the ways in which the money raised in taxation is used by the State and also according to the detailed character of the scheme of taxes under which it is raised. A discussion of this matter would require a volume to itself. In *A Study in Public Finance* some aspects of it are investigated. It lies outside our present scope.

CHAPTER XLVIII

THE PASSAGE TO REAL LIFE

§ 1. Our study of the last and most complex of the four models distinguished in Chapter XXXVI., and, therewith, our study of stationary states, is now accomplished. There only remains the epilogue. Let us imagine a world in which the governing conditions stand stationary for long periods together, while on rare occasions disturbances occur, each of which always works itself out completely before a new disturbance takes place. The economics of that world would comprise, besides the division dealing with stationary states, which has been the subject of this book, a second division dealing with the process of transition from one stationary state to another. When the governing conditions alter there is immediately set up a disequilibrium, or series of disequilibria, in such wise that persons and equipment engaged in one of the occupations find that they would be better off elsewhere—*i.e.* if converted from one form of sub-factor to another—and are thus under an urge towards movement; or—or, it may be, and—in such wise that certain factors of production find themselves in receipt of a rate of pay above or below the maintenance price of the existing quantity of them. The essential note of transition is the existence of these disequilibria. So soon as they have been corrected, the economics of transition withdraw into the background and the economics of stationary states resume control.

§ 2. No attempt, of course, will be made, in the last chapter of a book already long, to work out, even in the sketchiest way, an economics of transitions. It is important, however, to understand the general character of the disequilibria which

rule in them. Whether it is the distribution of a factor among sub-factors, or the quantity of a fundamental factor, that the emerging new stationary state requires to be different from what it was, there is in each instant of the transition a kind of subordinate equilibrium. When it is the distribution of a factor among sub-factors that is being altered, at each instant every sub-factor is paid at a rate equal to the value of its marginal private product—or at that rate modified to allow for monopolistic activities by entrepreneurs; but the fundamental factor, out of which the several sub-factors are made, is under urge to move out of one sub-factor into another. When it is the quantity of a fundamental factor that is being altered, that factor again is paid at every instant the value of the marginal private product—or at that rate appropriately modified—of the quantity of it which then exists. Moreover, the rate of inflow of new increments of the factor into existence, or its rate of outflow out of existence, is throughout in balance, in the sense that the value of its marginal private product (the demand price) is equal to the supply price of the stream that is flowing in or of the stream that is flowing out. Thus suppose that the change in the governing conditions is of such a kind that the stock of capital must be enlarged, but the stock of labour remains unaltered. The immediate consequence of a change of this kind is—in accordance with what lies behind the change—either that a higher rate of interest is offered in respect of nil new investment, or that people become willing to undertake some new investment in response to the existing rate of interest. In either event, both immediately and over the whole period of transition, the annual (or weekly) demand for and the annual (or weekly) supply of new investment at the price actually existing are equal to one another. At each instant, we may suppose, everybody is balancing desires and aversions at the margin, and, unhampered by any friction, does what it pays him best to do. Demand and supply are adjusted at every instant in respect of rates of addition to capital stock. The same thing is true when the new set of governing conditions requires an enlargement in the stock of labour alone or in the stocks both of labour and of capital equipment. In either event, at

each instant, the existing state of either stock being given, demand and supply, in respect of the rate of addition to the other, are in adjustment. In this sense—a sense in which some economists sometimes use the term—there is equilibrium. But demand and supply are not adjusted in respect of the standing quantities of the stocks of capital or/and labour. In this deeper sense the system is in disequilibrium, and must remain so until the new stationary state is established and additions to stock are no longer being made. The same thing is, of course, true when the change in governing conditions is such that a reduction in one or both stocks is required.

§ 3. In the kind of world we are here contemplating the comparative importance for practice of the economics of stationary states, as against states of transition, depends partly upon the length of the intervals between successive disturbances in the governing conditions and partly upon the speed with which the transition from that stationary state proper to the old to that proper to the new conditions is accomplished. The shorter the interval and the slower the speed of adjustment, the more important are the economics of transition. The length of the interval must, from our present standpoint, be accepted as sheer matter of fact, about which nothing of interest can be said; but the influences which determine the speed of adjustment require a brief notice.

§ 4. When the adjustment to be made has to do with the distribution of a fundamental factor among sub-factors the time needed for it depends upon two things. For two separate processes are set in motion. On the one hand, capital is transferred, *e.g.* from armaments to textiles, by new textile equipment being built instead of existing armaments equipment being maintained intact ; and labour is transferred out of coal-miners into shipbuilders by a deflection of the stream of new recruits that is flowing continuously into industry. On the other hand, existing adult workpeople and existing items of capital equipment are moved bodily out of one sub-factor and converted by training or manipulation into another. Adjustments of the first kind cannot, in their nature, be made instantaneously. On the contrary, even in a perfectly

lubricated and absolutely frictionless world they are bound to be spread over an interval of time. How long this interval is depends on the rate of turnover, so to speak, of the fundamental factor in relation to the amount of transfer that has to be effected. Thus suppose that it is required to raise the number of coal-miners from x to $(x+a)$ and to decrease the number of shipbuilders from y to $(y-a)$; and that in either occupation the rate at which employees die or retire is $2\frac{1}{2}$ per cent per annum of the original number. This adjustment, if no mistakes are made and no friction intervenes, cannot take less time than 40 a/y years; and in fact it will take substantially longer than that. An exactly analogous proposition is true of transfers of capital out of one kind of equipment into another. It will be observed that the proportionate change to be effected in the sub-factor to be *expanded* has no significance. What is relevant is the proportionate change required in the one to be *contracted.* A much longer time is required to transfer a given number of men or quantity of capital from a small sub-factor to a large one than *vice versa.* When, therefore, a shift in the governing conditions requires a particular sub-factor to be expanded at the expense of the whole body of sub-factors made out of the same fundamental factor—in general a much larger mass—it will usually be possible for a large percentage expansion to take place in a fairly short time. This conclusion is confirmed by the fact that in real life new industries resulting from specific inventions and requiring new types of skill (*e.g.* that of the motor mechanic) or of machines are found to grow up very rapidly. But there are also adjustments of the second of the two kinds distinguished at the beginning of this section. The speed with which these can be accomplished is in suitable conditions very high. Thus, though some sorts of fixed capital are highly specialised, other sorts, *e.g.* some buildings, can be adjusted fairly easily to new uses, while a large proportion of working capital—materials and intermediate goods—is not specialised at all. Similar considerations hold good of labour. Marshall writes on this matter as follows: "The movements of adult labour from trade to trade and from place to place can in some cases be

so large and so rapid as to reduce within a very short compass the period which is required to enable the supply of labour to adjust itself to the demand. That general ability, which is easily transferable from one trade to another, is every year rising in importance relatively to that manual skill and technical knowledge which are specialised to one branch of industry. And thus economic progress brings with it, on the one hand, a constantly increasing changefulness in the methods of industry, and therefore a constantly increasing difficulty in predicting the demand for labour of any kind a generation ahead; but, on the other hand, it brings also an increasing power of remedying such errors of adjustment as have been made."[1] Nothing can usefully be added to that passage.

§ 5. Turn next to adjustments in the stocks of the two fundamental factors, capital in general and labour in general. These two sorts of adjustment appear on the surface to be accomplished in quite different ways. Thus the stock of capital equipment is increased, and can only be increased, in any year by a portion of the real income of that year, *i.e.* of the net product after existing capital stock has been maintained intact, being withdrawn from consumption and given the form of new capital equipment. There does not seem to be anything analogous to this as regards labour. The apparent discrepancy, however, is due solely to the fact that resources devoted to building up additions to the stock of labour rank *by definition* as resources devoted to consumption, and not as withdrawn from consumption. But these resources are in fact withdrawn from the consumption of people of working age; and, only in so far as they are thus withdrawn, can additions to the stock of the working population be built up. There is thus a true parallelism between equipment growth and labour growth. Nevertheless, it is convenient to consider accretions to these two dominant factors separately.

§ 6. The speed with which a given proportionate adjustment in the stock of capital can be made—I confine attention to upward adjustments—depends principally upon the com-

[1] *Principles*, p. 657.

parative sizes of the stock of capital in existence and of annual real income. When the stock is small and the real income large, a given proportionate change in the stock will obviously be made, in response to a given inducement, much more rapidly than is possible in converse conditions. The speed of adjustment also depends in part on how long the particular elements of capital, whose quantity it is desired to alter, take to create. Moreover, it is highly relevant to know whether the real demand for capital is elastic or inelastic; for, the more elastic it is, the larger will be the increase of stock that is needed in consequence of a given upswing in demand, and so of a given immediate increase in the rate of pay offered per unit to the original stock of capital. It is thus impossible to make any estimate of speed that is of general application. With a situation as regards real income, quantity and nature of capital stock and elasticity of demand for capital roughly similar to the situation in England now, there is, I think, little doubt that the adjustment required to fit, say, a five per cent upswing in the demand for capital would occupy, in an isolated community, a good many years. No doubt, in a community that was able to import capital from abroad, as well as to create it for itself, greater speed could be attained, provided always that the upswing of demand was confined to that country.[1]

§ 7. Let us turn now to labour. Here there are two dominant considerations. The first is that in any normally constituted population the stock of persons of working age, say, from 15 to 64, is very large relatively to the number of new persons who come to working age annually. In England and Wales at the 1931 census the number aged 15-64 was 30½ million, and the number aged 14 was 627,000. Even, therefore, if it were possible for the annual number of new entrants into working age to be altered instantly at will, an enormous proportionate change in that number would be needed to bring about quickly a comparatively small proportionate change in the stock of labour. To increase the stock by 5 per cent in one year the

[1] Obviously an analysis precisely similar to that given above holds good of stocks of durable *consumers'* goods, such as pianos, houses, automobiles and so on.

number of new entrants would need to be more than doubled. The second dominant consideration is even more significant. With capital equipment, though some sorts take a little time to make, yet the interval between turning resources towards building a capital instrument and the completion of that instrument is not often more than a year. Ships, large buildings and coal-mines are longer in gestation, but four or five years is enough even for them. With labour reactions from demand, though, of course, they can modify the efficiency and the intensity of work of existing labour fairly soon, cannot begin to operate on the numbers of working age—apart from possible reactions on the death-rate—till at least fourteen years have passed. Thus not only is there a considerable period, during which the process of adding to stock, once begun, is being carried through to completion, but, over and above this, there is a preliminary delay of at least fourteen years before action directed to the process can make the process begin.

§ 8. These considerations suggest that, for some very important sorts of change, the run needed to pass from the stationary state proper to one set of governing conditions to that proper to another set is a very long one. Hence the economics of transition would play a large part even in a world where the governing conditions changed but rarely. In the actual world, since in fact these conditions change frequently and widely, they play a still larger part. Moreover, in this world the new stationary state, to which a given shift in the governing conditions point, is never attained in fact; for some new disturbance always intervenes. Thus its history is not made up of periods of stationariness, to which the economics of stationary states is fully applicable, and periods of transition. Transition rules always; stationariness never; the long run never comes. It follows that the analysis worked out in this book cannot by itself make any large direct contribution to the study of real life. It provides a taking-off place, but little more; a first stage only, which needs extensive supplement. The building is much more than the foundation. But, none the less, to take pains over the foundation is not to waste time.

APPENDICES

I

Appendix to Chapter I

The state contemplated by Marshall in the passage cited in § 5 of Chapter I., put in a rigorous form, is one in which technique is everywhere constant, the quantities of all the factors of production are increasing in equal proportions, no economies or diseconomies of large scale prevail anywhere, and people's tastes are constant. In such a state the consumption and production per head of everything and the proportion of people in different occupations will be constant, while the quantity of trade expands proportionately with population. At first sight it might seem that these conditions can be satisfied irrespective of the rate at which the economic cosmos is expanding—equally whether the expansion is steady or irregular. It is not difficult, however, to show that this is not so. A given rate of pay for the services of the factor capital, which, if we wish, can be expressed as a rate of interest, as the number of people with given incomes grows, necessarily evokes new investment, or a net creation of new capital per unit of time, that grows in the same proportion as the number of people. But the stock of capital is, *ex hypothesi*, growing in the same proportion as the number of people. Therefore the proportionate rate of growth of the annual (or daily) addition made to capital is equal to the proportionate rate of growth of the total stock. If we write $f(t)$ for the total stock at time t, this means that $\dfrac{f'(t)}{f(t)} = \dfrac{f''(t)}{f'(t)}$ for all values of t. But this implies that

$$\frac{f'(t)}{f(t)}\left\{\frac{f''(t)}{f'(t)} - \frac{f'(t)}{f(t)}\right\} = 0,$$

i.e. that

$$\frac{d}{dt}\left\{\frac{f'(t)}{f(t)}\right\} = 0.$$

That is to say, $f'(t)/f(t)$ is constant; *i.e.* the stock of capital and, therefore, of course, also the stocks of all the other factors, increase at a constant geometrical rate. It is only with an economic cosmos expanding in this manner that the conditions envisaged by Marshall can be satisfied.

II

Appendix to Chapter VIII

Substitutes and Complements.

If r_1 and r_2 be the quantities of the two commodities, the desire for the r_1th unit of the first is, in general, $\phi_1(r_1, r_2)$, and for the r_2th unit of the second $\phi_2(r_1, r_2)$. With commodities that, in respect of extant quantities, are neither substitutes nor complements, both $\partial\phi_1/\partial r_2$ and $\partial\phi_2/\partial r_1 = 0$. With commodities that are mutual substitutes in any degree both are negative; with commodities that are mutual complements both are positive. These are the definitions currently accepted among economists. It has been suggested by Dr. Hicks, basing himself on Pareto, that they are ambiguous. All that is meant, however, is that the values $\partial\phi/\partial r$ cannot be unambiguously inferred from a knowledge of people's scales of preference and indifference as between related commodities; for the reason that ϕ itself cannot be so inferred. If we *postulate* ϕ to start with, there is no difficulty.

III

Appendix to Chapter IX

The differences between quantities of work done in different conditions in a Crusoe economy.

Problem 1.—When Robinson produces only one kind of commodity, to find the effect on the quantity of work done by him of a small equiproportional increase in his desire schedule for that commodity.

Let his marginal aversion from work be $f(x)$, the output of x work $F(x)$, and his marginal desire for product $c\phi\{F(x)\}$. Then

$$f(x) = cF'(x) . \phi\{F(x)\} \qquad . \quad . \quad . \quad (1)$$

When a small equiproportional increase in desire takes place, c (which may be put equal to 1 in the initial situation) becomes $(c+dc)$. We then have

$$\frac{d}{dc}f(x) = \frac{d}{dc}\Big[cF'(x) . \phi\{F(x)\}\Big] \qquad . \quad . \quad . \quad (2)$$

Write e for Robinson's elasticity of marginal aversion from work, η for his elasticity of marginal desire for the commodity, and ϵ for his elasticity of marginal productivity.

Then

$$\frac{1}{e} = \frac{xf'}{f},$$

$$\frac{1}{\eta} = \frac{F\phi'}{\phi},$$

$$\frac{1}{\epsilon} = \frac{xF''}{F'}.$$

From equations (1) and (2) we then obtain

$$\frac{dx}{dc} = \frac{-xe\eta}{e\frac{xF'}{F} - \eta + \frac{e\eta}{\epsilon}} \qquad . \quad . \quad . \quad . \quad (A)$$

We know that e is positive and η negative. Since, as set out

in the text of Chapter IX., we are explicitly excluding increasing returns, ϵ is either nil or negative. Hence the denominator of this expression, and so the expression as a whole, is necessarily positive. Moreover differentiation shows that it is larger, the larger are $e, -\eta$ and $-\epsilon$, and the smaller is F'/F.

In the special case of constant returns, *i.e.* where $\epsilon = -\infty$, equation (A) reduces to

$$\frac{dx}{dc} = -\frac{xe\eta}{e-\eta}.$$

Problem 2.—When Robinson produces only one kind of commodity, to find the effect on the quantity of work done by him of a small equiproportional improvement in his productivity.

Let us now write $cF(x)$ for the output of x work and $\phi\{cF(x)\}$ for Robinson's marginal desire for product. Then

$$f(x) = cF'(x) . \phi\{cF(x)\} \quad . \quad . \quad . \quad (1)$$

When a small equiproportional improvement in productivity takes place, c becomes $(c + dc)$. We then have

$$\frac{d}{dc}f(x) = \frac{d}{dc}\Big[cF'(x) . \phi\{cF(x)\}\Big] \quad . \quad . \quad . \quad (2)$$

From equations (1) and (2) we obtain

$$\frac{dx}{dc} = \frac{-xe(1+\eta)}{e\frac{xF'}{F} - \eta + \frac{e\eta}{\epsilon}}. \quad . \quad . \quad . \quad (B)$$

Since, as we have seen, e is positive, η negative and ϵ either negative or nil, the denominator of this expression is necessarily positive. It follows that the numerator, and so the whole expression, is positive or negative, according as $-\eta >$ or < 1. Further, we find by differentiation that the expression is larger absolutely, *i.e.* is a larger positive quantity if positive and a smaller negative quantity if negative, the larger is $-\eta$. If $-\eta > 1$, it is a larger positive, if $-\eta < 1$, a larger negative, quantity, the larger are e and $-\epsilon$, and the smaller is F'/F.

In the special case of constant returns, *i.e.* when $\epsilon = -\infty$, the above expression reduces to

$$\frac{dx}{dc} = -\frac{xe(1+\eta)}{e-\eta}.$$

Problem 3.—When Robinson is producing two independent commodities (*i.e.* commodities that are neither substitutes nor complements), to determine the effect on the quantity of work done, (1) on both commodities together and (2) on the one directly affected, by a small equiproportional increase in his desire for one of them.

Let x be the total quantity of Robinson's work; $k_a x$ the quantity done on commodity A and $k_b x$ that done on commodity B, so that $k_a + k_b = 1$; $F_a(k_a x)$ and $F_b(k_b x)$ the quantities of these commodities; $c\phi\{F_a(k_a x)\}$ and $\psi\{F_b(k_b x)\}$ Robinson's marginal desires for them; and $f(x)$ his marginal aversion from work, which is assumed to depend on aggregate quantity of work only, without regard to its distribution. Let the increase of desire be in respect of commodity A. Then, on the plan followed above, we have the equations:

$$f(x) = c\frac{dF_a(k_a x)}{d(k_a x)} \cdot \phi\{F_a(k_a x)\}, \quad . \quad . \quad . \quad (1)$$

$$\frac{d}{dc}f(x) = \frac{d}{dc}\left\{c\frac{dF_a(k_a x)}{d(k_a x)} \cdot \phi\{F_a(k_a x)\}\right\}, \quad . \quad . \quad (2)$$

$$f(x) = \frac{dF_b(k_b x)}{d(k_b x)} \cdot \psi\{F_b(k_b x)\}, \quad . \quad . \quad . \quad (3)$$

$$\frac{d}{dc}f(x) = \frac{d}{dc}\left\{\frac{dF_b(k_b x)}{d(k_b x)} \cdot \psi\{F_b(k_b x)\}\right\}, \quad . \quad . \quad (4)$$

Write e, as before, for Robinson's elasticity of marginal aversion from work; η_a and η_b for the elasticities of his marginal desire for commodities A and B, and ϵ_a, ϵ_b for the elasticities of marginal productivity of work on A and B respectively. Also write

$$\left\{\frac{e\eta_a}{\epsilon_a k_a} + xe\frac{F_a'}{F_a}\right\} = \frac{1}{m}$$

$$\left\{\frac{e\eta_b}{\epsilon_b k_b} + xe\frac{F_b'}{F_b}\right\} = \frac{1}{n}.$$

Solving the above equations we have:

$$\frac{dx}{dc} = \frac{-mxe\eta_a}{1 - m\eta_a - n\eta_b}, \quad . \quad . \quad . \quad (C)$$

$$\frac{d(k_a x)}{dc} = \frac{-mxe\eta_a(1 - n\eta_b)}{1 - m\eta_a - n\eta_b}. \quad . \quad . \quad . \quad (D)$$

Differentiation shows that dx/dc is larger, the larger are $e, -\eta_a$, $-\epsilon_a$ and k_a, and the smaller are $-\eta_b$, $-\epsilon_b$ and k_b.

In the special case where both commodities are produced under conditions of constant returns $\frac{1}{m} = \frac{1}{n} = e$.

Hence
$$\frac{dx}{dc} = \frac{-xe\eta_a}{e - \eta_a - \eta_b},$$

$$\frac{d(k_a x)}{dc} = \frac{-x\eta_a(e - \eta_b)}{e - \eta_a - \eta_b}.$$

These values are, it will be observed, independent of the ϵ's and the k's.

In the still more special case where also $\eta_a = \eta_b = \eta$, the above equalities reduce to

$$\frac{dx}{dc} = \frac{-xe\eta}{e-2\eta},$$

$$\frac{d(k_a x)}{dc} = \frac{-x\eta(e-\eta)}{e-2\eta}.$$

Problem 4.—When Robinson is producing two independent commodities, to determine the effect on the quantity of work done, (1) on both commodities together and (2) on the one directly affected, of a small equiproportional improvement in productivity in respect of one of them.

By analogy with Problem 2, the relevant equations now are

$$f(x) = c\frac{d\{F_a(k_a x)\}}{d(k_a x)}.\phi\{cF_a(k_a x)\}, \quad \quad (1)$$

$$\frac{d}{dc}f(x) = \frac{d}{dc}\left\{c\frac{d\{F_a(k_a x)\}}{d(k_a x)}.\phi\{cF_a(k_a x)\}\right\}, \quad \quad (2)$$

$$f(x) = \frac{dF_b(k_b x)}{d(k_b x)}.\psi\{F_b(k_b x)\}, \quad \quad (3)$$

$$\frac{d}{dc}f(x) = \frac{d}{dc}\left\{\frac{dF_b(k_b x)}{d(k_b x)}.\psi\{F_b(k_b x)\}\right\}. \quad \quad (4)$$

Solving these equations, we obtain, with the notation of Problem 3,

$$\frac{dx}{dc} = \frac{-mxe(1+\eta_a)}{1-m\eta_a-n\eta_b}, \quad \quad (E)$$

$$\frac{d(k_a x)}{dc} = \frac{-mxe(1+\eta_a)(1-n\eta_b)}{1-m\eta_a-n\eta_b}.$$

As in Problem 3, differentiation shows that dx/dc is larger, the larger are e, $-\eta_a$, $-\epsilon_a$ and k_a, and the smaller are $-\eta_b$, $-\epsilon_b$ and k_b. When $-\eta_a < 1$ for dx/dc to be larger means, of course, that it is a smaller negative quantity.

In the special case where both commodities are produced under conditions of constant returns,

$$\frac{dx}{dc} = \frac{-xe(1+\eta_a)}{e-\eta_a-\eta_b},$$

$$\frac{d(k_a x)}{dc} = \frac{-x(1+\eta_a)(e-\eta_b)}{e-\eta_a-\eta_b}.$$

As in Problem 3, these values are independent of the ϵ's and the k's.

In the still more special case where also $\eta_a = \eta_b = \eta$,

$$\frac{dx}{dc} = \frac{-xe(1+\eta)}{e-2\eta},$$

$$\frac{d(k_a x)}{dc} = \frac{-x(1+\eta)(e-\eta)}{e-2\eta}.$$

Problem 5.—When Robinson is producing only one kind of commodity, to prove that an improvement in his productivity, whether equiproportional or not, that conforms to the conditions of Chapter IX. § 3, always leads to an increase in output.

Write x for the original quantity of work, $F(x)$ for the original product of x work, $\phi\{F(x)\}$ for Robinson's desire of the marginal unit of product, and $f(x)$ for his aversion from the marginal unit of work.

Then $$F'(x) . \phi\{F(x)\} = f(x).$$

Write $(x+h)$ for the quantity of work after the improvement and $\psi(x+h)$ for the associated quantity of product.

Then $$\psi'(x+h) . \phi\{\psi(x+h)\} = f(x+h).$$

Our data are that $\psi(x) > F(x)$ and $\psi'(x) > F'(x)$ for all values of x. We also know that ϕ and f are positive; while ϕ' is negative and f' positive; and, since we are postulating constant or diminishing returns, that F'' and ψ'' are nil or negative. If h is positive, since $\psi(x+h)$ is by definition $> F(x+h)$, it is *a fortiori* $> F(x)$.

If h is negative, since f' is positive, it follows from the above equation that

$$\psi'(x+h) . \phi\{\psi(x+h)\} < F'(x) . \phi\{F(x)\}.$$

But, if h is negative, $\psi'(x+h)$, which is $> \psi'(x)$, is $> F'(x)$.

Therefore $$\phi\{\psi(x+h)\} < \phi\{F(x)\}$$
$$\therefore \ \psi(x+h) > F(x).$$

That is to say, whether quantity of work is increased or diminished, quantity of product *must* be increased.

Problem 6.—When Robinson is producing two independent commodities A and B, to prove that an improvement in his productivity in respect of A need not always lead to an increase in the output of A.

Let us consider an equiproportional improvement in the case of constant returns. In the notation of Problem 4, the quantity of commodity A that is produced is then $cqk_a x$, where q is a constant. This quantity is increased or not by an equiproportional improvement according as $\frac{d}{dc}\{cqk_a x\}$ is positive or negative.

Now $$\frac{d}{dc}\{cqk_ax\} = q\left\{k_ax + c\frac{dk_ax}{dc}\right\}.$$

But we found in Problem 4 that

$$\frac{d(k_ax)}{dc} = \frac{-x(1+\eta_a)(e-\eta_b)}{e-\eta_a-\eta_b},$$

$$\therefore \frac{d}{dc}\{cqk_ax\} = qx\left\{k_a - c\frac{(1+\eta_a)(e-\eta_b)}{e-\eta_a-\eta_b}\right\}.$$

Since we are putting c in the initial situation $=1$, this is equal to

$$\frac{qx}{e-\eta_a-\eta_b}\left\{k_a(e-\eta_a-\eta_b) - (1+\eta_a)(e-\eta_b)\right\}.$$

Now the portion of this expression outside the brackets is positive; and, if $-\eta_a > 1$, the portion inside is positive also. Hence in these conditions $\frac{d}{dc}\{cqk_ax\}$ cannot be negative. But, if $-\eta_a < 1$, the expression $-(1+\eta_a)(e-\eta_b)$ is negative. For sufficiently small values of k_a, therefore, $\frac{d}{dc}\{cqk_ax\}$ will be negative. That is to say, the quantity of commodity A that is produced *may* be diminished in consequence of an improvement in respect of its production.

IV

Appendix to Chapter XVI

The relation between value of marginal product and marginal product of value to an isolated producer.

Write x for the total number of units of work engaged on his product by a particular producer; $F(x)$ for this producer's output of commodity; and $\phi\{F(x)\}$ for the demand price per unit of $F(x)$ units of product provided by him. The *value of* our producer's *marginal product* is then $F'(x).\phi\{F(x)\}$.

The *marginal product of value* to him

$$= F'(x)\left\{\phi\{F(x)\} + \frac{d\phi\{F(x)\}}{dF(x)} \cdot F(x)\right\}$$

$$= F'(x).\phi\{F(x)\}\left\{1 + \frac{d\phi F(x)}{dF(x)} F(x) \div \phi\{F(x)\}\right\}.$$

Write η for the elasticity of the demand for the product from our producer.

Then $$\frac{1}{\eta} = \left\{\frac{d\phi\{F(x)\}}{dF(x)} \cdot F(x) \div \phi\{F(x)\}\right\}.$$

$\therefore$ marginal product of value of his work to our producer

$$= F'(x).\phi\{F(x)\}\left\{1 + \frac{1}{\eta}\right\}$$

$=$ value of marginal product of that work multiplied by $\left\{1 + \frac{1}{\eta}\right\}$.

V

Appendix to Chapter XVII

Monopoly in relation to Mathematical Determinateness.

Case 1. Trade between two monopolists.

The argument of § 2 of Chapter XIX. can be expressed more exactly in symbols. Let Robinson exchange x nuts against Jones's y apples. Then Robinson's net satisfaction from the deal may be written $\phi(x, y)$ and Jones's $\psi(x, y)$. With Robinson and Jones both seeking to maximise their respective satisfactions we have

$$\frac{d}{dx}\phi(x, y) = 0,$$

$$\frac{d}{dy}\psi(x, y) = 0.$$

These equations may be written

$$\frac{\partial}{\partial x}\phi(x, y) + \frac{dy}{dx} \cdot \frac{\partial}{\partial y}\phi(x, y) = 0,$$

$$\frac{\partial}{\partial y}\psi(x, y) + \frac{dx}{dy} \cdot \frac{\partial}{\partial x}\psi(x, y) = 0.$$

We thus appear to have two equations and two unknowns, and so a determinate solution. But dy/dx is the change in Jones's offer of apples that Robinson *expects* will result from an increase by dx in his own offer of nuts: and dx/dy is the corresponding change that Jones *expects* from an increase of dy in his offer. These two quantities are related in a precise manner to their respective expectations about the effect on the rate of exchange; for $\frac{d}{dx}\left\{\frac{y}{x}\right\} = \frac{1}{x}\left(\frac{dy}{dx} - \frac{y}{x}\right)$. The problem is or is not determinate according as these expectations are or are not given.

First, if both Robinson and Jones act monopolistically, neither dy/dx nor dx/dy are given. There appear to be two unknowns and two independent equations. But in fact there are four unknowns. Therefore the system is indeterminate.

Secondly, if, while Jones acts monopolistically, Robinson acts in accordance with the rule of competition, this implies that he expects the effect of a change in his offer on the rate of exchange to be nil. Therefore for him $\frac{d}{dx}\left\{\frac{y}{x}\right\}=0$. This implies $\frac{dy}{dx}=\frac{y}{x}$. Therefore Robinson's equation becomes

$$\frac{\partial}{\partial x}\{\phi(x, y)\}+\frac{y}{x}\frac{\partial}{\partial y}\{\phi(x, y)\}=0.$$

That is to say, y is a function of x in such wise that the bargain is bound to lie on some part of Robinson's demand curve. Thus dx/dy, *i.e.* the change in x that will, as a matter of objective fact, be associated with a small change in y, is a function of x and y. But in these conditions Jones's view adjusts itself to the fact. Therefore the dx/dy which signifies Jones's view of the effect of a small change in y on x is identical with the dx/dy which signifies the actual effect of this. Thus this element in Jones's equation is also a function of x and y. There are in fact, as well as in appearance, only two unknowns; and the system is determinate.

Thirdly, if Robinson and Jones both act competitively, Robinson's equation is, as above,

$$\frac{\partial}{\partial x}\{\phi(x, y)\}+\frac{y}{x}\frac{\partial}{\partial y}\{\phi(x, y)\}=0$$

and Jones's in like manner

$$\frac{\partial}{\partial y}\{\psi(x, y)\}+\frac{x}{y}\frac{\partial}{\partial x}\{\psi(x, y)\}=0.$$

These two equations obviously contain only two unknowns, and are, therefore, sufficient to determine x and y. They can be written

$$\left\{\frac{\partial}{\partial x}\{\phi(x, y)\}\div\frac{\partial}{\partial y}\{\phi(x, y)\}\right\}=-\frac{y}{x},$$

and

$$\left\{\frac{\partial}{\partial y}\{\psi(x, y)\}\div\frac{\partial}{\partial x}\{\psi(x, y)\}\right\}=-\frac{x}{y}.$$

These expressions are a generalised equivalent of Jevons's famous equations for isolated, but representative, persons trading together.

Case 2. Trade of two monopolists producing the same commodity with a third party who acts on the rule of competition.

Let us write x_1 for the quantity of work that Jones A sells, and x_2 analogously for Jones B. Write ϕ for the demand function of the market; F_1, F_2 for the cost functions in terms of aversions; and

ψ_1 and ψ_2 for the desire functions of Jones A and Jones B. Then the condition for Jones A's satisfaction, as defined in Chapter I. § 4, to be maximised, is that

$$\frac{d}{dx_1}\left\{\psi_1\{x_1\phi(x_1+x_2)\}\right\} - F'(x_1) = 0.$$

That is

$$\frac{\partial}{\partial x_1}\left\{\psi_1\{x_1\phi(x_1+x_2)\}\right\} + \frac{dx_2}{dx_1}\cdot\frac{\partial}{\partial x_2}\left\{\psi_1\{x_1\phi(x_1+x_2)\}\right\} - F'(x_1) = 0,$$

where dx_2/dx_1 and dx_1/dx_2 signify respectively Jones A's opinion of the effect of a small increase of x_1 on x_2, and Jones B's opinion of the effect of a small increase of x_2 on x_1. The condition for Jones B is analogous. The situation is thus determinate for any assigned values of dx_2/dx_1 and dx_1/dx_2. Cournot's assumption is that in fact

$$\frac{dx_2}{dx_1} = \frac{dx_1}{dx_2} = 0.$$

The other assumption, considered in Chapter XVII. § 5, is that

$$\frac{dx_2}{dx_1} = \frac{x_2}{x_1} \quad \text{and} \quad \frac{dx_1}{dx_2} = \frac{x_1}{x_2}.$$

It is argued in the text that neither of these assumptions is, in general, warranted.

Rider on Case 2. The range of indeterminateness in the foregoing case when the problem is indeterminate.

This range can be found by considering what are the extreme values possible for dx_2/dx_1 and dx_1/dx_2. It seems clear that these expressions cannot respectively have greater positive values than x_2/x_1 and x_1/x_2. That is to say, one Jones cannot reckon that, by decreasing his output in a given proportion, he will cause the other Jones to decrease his in a larger proportion. This implies that the smallest aggregate output of the two Joneses together that is possible under duopoly is an output equal to what they would have produced had they been combined together in a single monopoly. At the other extreme limit each may hold that a given cut in his output will be associated with an equivalent expansion in the other's output; *i.e.* that

$$\frac{dx_2}{dx_1} = -1$$

and likewise $$\frac{dx_1}{dx_2} = -1.$$

Under this arrangement Jones A's equation degrades to

$$\frac{d\psi_1\{x_1\phi(x_1+x_2)\}}{d\{x_1\phi(x_1+x_2)\}}\cdot\phi(x_1+x_2)-\mathrm{F}'(x_1)=0;$$

and similarly with Jones B. This gives competitive equilibrium. Hence the limits of indeterminateness are the outputs proper respectively to a single unified monopoly and to competition.

VI

Appendix to Chapter XX

The conditions in which in an assembly of Robinsons, each specialised to a different commodity, the substitution of monopolistic action for competitive action by one of them decreases or increases the aggregate quantity of work done.

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§ 1. It is postulated that: (i) all the Robinsons operate under conditions of constant return ; (ii) initially the output of a unit of work by each Robinson is one unit of product and is equal in value to the output of a unit of work by each other Robinson : (iii) initially the particular Robinson A, who contemplates monopolistic action, consumes practically none of his own commodity, but sells the whole of it; and (iv) the relative values of the products of all the Robinsons other than Robinson A are the same after as before the change.

§ 2. Initially let x be the output of Robinson A due to x units of work and $\phi(x)$ the demand price for it in terms of other people's commodities. The aggregate of all work in making Robinson A's product and in making the goods exchanged against it is then $\{x + x\phi(x)\}$. The condition for this to increase as x diminishes is that $\frac{d}{dx}x + \{x\phi(x)\}$ is negative.

Write η_a for the elasticity of demand for Robinson A's product in respect of output x. Then this condition may be written that $\left\{\frac{1}{\phi(x)} + 1 + \frac{1}{\eta_a}\right\}$ is negative.

But $x\phi(x) = x$. Hence the required condition is that $\left(2 + \frac{1}{\eta_a}\right)$ is negative, *i.e.* that $-\eta_a < \frac{1}{2}$. Unless this condition is satisfied for some substantial part of the range of Robinson A's output lying between that proper to competition and that proper to monopoly, the amount of work done by Robinson A *plus* the amount done by the other Robinsons for sale to him cannot be increased in consequence of monopolistic action by Robinson A.

§ 3. The changes in quantity of work done by the other Robinsons for exchange against Robinson A's product in consequence of Robinson A turning from competitive to monopolistic action is not, in general, the same as the change in the total quantity of work done by them. Let $\frac{(n-1)}{n}$ be the proportion of the other Robinsons' work that in the initial situation was engaged in providing for them things other than Robinson A's product. Let the elasticity of a representative other Robinson's marginal aversion from work be e and the elasticity of his marginal desire for the other things—which, from assumption iv of § 1, is the same for each kind of other thing—be ϵ_b. In consequence of Robinson A's turning to monopoly let h more (or less) of the other Robinsons' work be devoted to making purchases from him. Write q for the total increment (or decrement) of work performed by these other Robinsons. It follows that

$$q\frac{1}{e} = (q-h)\frac{1}{\epsilon_b}\frac{(n-1)}{n}.$$

Therefore, when n is large,

$$q = (\text{approximately})\ h\frac{e}{e-\epsilon_b}.$$

Thus, if either $e=\infty$, or $\epsilon_b=0$, $q=h$. In all other circumstances q must be greater than h. Since e is obviously positive and ϵ_b negative, q falls short of h in a larger proportion the smaller is e and the larger is $-\epsilon_b$.

§ 4. It follows that, unless either $e=\infty$ or $\epsilon_b=0$, even when the conditions are such that the quantity of work done by Robinson A *plus* the quantity done by the other Robinsons to exchange with Robinson A is increased by Robinson A's turning to monopoly, the quantity of work done by Robinson A *plus* the quantity done by the other Robinsons for all purposes, *i.e.* the aggregate quantity of work done by all the Robinsons together, need not be increased. The condition for this aggregate quantity to be increased is evidently that

$$\frac{d}{dx}\left\{x + \frac{e}{e-\epsilon_b}x\phi(x)\right\}$$

is negative; *i.e.* that $\left\{\frac{1}{\phi(x)} + \left(1+\frac{1}{\eta_a}\right)\frac{e}{e-\epsilon_b}\right\}$

is negative; *i.e.*, since $\phi(x)=1$, that $-\eta_a$ is less than $\frac{e}{2e-\epsilon_b}$. If ϵ_b is numerically large relative to e, this is substantially less than $\frac{1}{2}$; if, for example, $-\epsilon_b=e$, it is equal to $\frac{1}{3}$.

§ 5. In this formulation η_a, the elasticity of demand for Robinson A's commodity in terms of other commodities, whose relative values are assumed to be constant, is not, of course, independent of a representative other Robinson's elasticity of marginal aversion from work and marginal desire for commodities other than Robinson A's commodity, namely, e and ϵ_b. As before, let us suppose that $(n-1)$ times as much of the other Robinsons' work is devoted to other uses as is devoted to buying Robinson A's commodity; and let us write ϵ_a for the elasticity of a representative other Robinson's desire for Robinson A's commodity. On the assumption that constant returns rule everywhere, it can then be shown that

$$\eta_a = \epsilon_a \frac{en - \epsilon_b(n-1) + 1}{en - \epsilon_b(n-1) - \epsilon_a}.$$

Thus, provided that $-\epsilon_a = 1$, $\eta_a = \epsilon_a$ in all circumstances. If $-\epsilon_a < 1$, $-\eta_a > -\epsilon_a$; and conversely. As either n or e approaches infinity, η_a approaches ϵ_a, whatever the values of the other terms in the above expression.

VII

Appendix to Chapter XXI

Various conditions of interchange (in accordance with the rule of competition) in relation to the aggregate quantity of work done.

Problem 1.—Robinson and Jones are each specialised to the production of a single commodity, Robinson to A, Jones to B, both act in accordance with the rule of competition, and for both conditions of constant return prevail. Each purchases the whole of the other's output, and, in the initial situation, the quantities of work which they respectively do are equal. To determine the effect on the aggregate quantity of work done by the two together of a (small) equiproportional increase in Jones's desire schedule for commodity A.

Write a for the original amount of work done by Robinson on commodity A and b for that done by Jones on commodity B. Let each unit of work of either produce one unit of commodity. Choose units such that in the initial situation $a=b$. Write e_a, e_b for Robinson's and Jones's respective elasticities of marginal aversion from work; η_a for the elasticity of marginal desire for commodity A; η_b for the elasticity of marginal desire for commodity B.

Write, further, $f_a(a)$ for Robinson's marginal aversion from work; $f_b(b)$ for Jones's marginal aversion from work; $c\phi_a(a)$ for Jones's marginal desire for commodity A; and $\phi_b(b)$ for Robinson's marginal desire for commodity B. Then, by definition,

$$\frac{1}{e_a}=\frac{af_a'(a)}{f_a(a)},$$

$$\frac{1}{e_b}=\frac{bf_b'(b)}{f_b(b)},$$

$$\frac{1}{\eta_a}=\frac{a\phi_a'(a)}{\phi_a(a)},$$

$$\frac{1}{\eta_b}=\frac{b\phi_b'(b)}{\phi_b(b)}.$$

We have initially the equations

$$f_a(a) = \frac{b}{a}\phi_b(b), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$f_b(b) = \frac{ca}{b}\phi_a(a). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When a small equiproportional increase in Jones's desire for A takes place, c (which initially may be put $=1$) becomes $(c+dc)$.

$\therefore$ we have the two further equations

$$\frac{d}{dc}\left\{f_a(a)\right\} = \frac{d}{dc}\left\{\frac{b}{a}\phi_b(b)\right\}, \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{d}{dc}\left\{f_b(b)\right\} = \frac{d}{dc}\left\{\frac{ca}{b}\phi_a(a)\right\}. \quad . \quad . \quad . \quad . \quad (4)$$

Solving these equations, we find

$$\frac{da}{dc} + \frac{db}{dc} = -a\eta_a\frac{e_b\{(1+\eta_b)e_a + (1+e_a)\eta_b\}}{(1+\eta_a)(1+\eta_b)e_a e_b - (1+e_a)(1+e_b)\eta_a\eta_b}.$$

In the special case where $e_a = e_b = e$ and $\eta_a = \eta_b = \eta$, this reduces to

$$\frac{da}{dc} + \frac{db}{dc} = -a\frac{e\eta}{e-\eta}.$$

Problem 2.—In the same conditions as above, to determine the effect on the aggregate quantity of work done by Robinson and Jones together of a (small) equiproportional improvement in productivity in respect of commodity A.

Everything is the same as before save that Robinson's output of commodity is now written ca instead of a and Jones's marginal desire for A is written $\phi_a(ca)$ instead of $c\phi_a(a)$.

Robinson's marginal aversion from the work required to produce the marginal unit of the ca units of commodity A that he is producing is $\frac{1}{c}f(a)$: and one unit of A exchanges for b/ca units of B. Hence the relevant equations now are

$$f_a(a) = \frac{b}{a}\phi_b(b), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$f_b(b) = \frac{ca}{b}\phi_a(ca), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{d}{dc}f_a(a) = \frac{d}{dc}\left\{\frac{b}{a}\phi_b(b)\right\}, \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{d}{dc}f_a(a) = \frac{d}{dc}\left\{\frac{ca}{b}\phi_a(ca)\right\}. \quad . \quad . \quad . \quad . \quad (4)$$

Solving these equations, we find

$$\frac{da}{dc}+\frac{db}{dc}=-a(1+\eta_a)\frac{e_b\{(1+\eta_b)e_a+(1+e_a)\eta_b\}}{(1+\eta_a)(1+\eta_b)e_ae_b-(1+e_a)(1+e_b)\eta_a\eta_b}.$$

In the special case where $e_a=e_b=e$ and $\eta_a=\eta_b=\eta$, this reduces to

$$\frac{da}{dc}+\frac{db}{dc}=-a\frac{e(1+\eta)}{e-\eta}.$$

Problem 3.—In the above conditions, to determine the sign of the change in the aggregate quantity of work done by Robinson and Jones together in the two preceding cases and the relation of its magnitude to the various elasticities.

Write M for $\left\{\frac{da}{dc}+\frac{db}{dc}\right\}$ when it is Jones's desire that is altered, and N for this magnitude when it is Robinson's productivity that is altered.

It is then evident from an inspection of the expressions found for $\left\{\frac{da}{dc}+\frac{db}{dc}\right\}$ in the two preceding problems that, in the general case, since e_a and e_b are positive, both M and N are positive provided that $-\eta_a$ and $-\eta_b$ are both >1. If $-\eta_a>1$ but $-\eta_b<1$, the denominators of the two expressions must be positive, but the numerators may be either positive or negative; and, if $-\eta_a$, as well as $-\eta_b$, <1, the denominators themselves may be of either sign. Hence, except when both $-\eta_a$ and $-\eta_b>1$, the signs of M and N may, according to the state of the relevant elasticities, be either positive or negative.

To determine the relation between the signs of M and N and these elasticities, write

$$\{(1+\eta_a)(1+\eta_b)e_ae_b-(1+e_a)(1+e_b)\eta_a\eta_b\}=\mathrm{D},$$

and differentiate M and N to e_a, e_b, $-\eta_a$ and $-\eta_b$ respectively.

We obtain

$$\frac{d\mathrm{M}}{de_a}=a\eta_a(1+\eta_b)e_b\eta_b\frac{(1+\eta_a)e_b+(1+e_b)\eta_a}{\mathrm{D}^2}, \quad (1)$$

$$\frac{dM}{de_b}=a\eta_a(1+e_a)\eta_a\eta_b\frac{(1+\eta_b)e_a+(1+e_a)\eta_b}{\mathrm{D}^2}, \quad (2)$$

$$\frac{d\mathrm{M}}{d(-\eta_a)}=(1+\eta_b)e_ae_b\frac{(1+\eta_a)e_b+(1+e_b)\eta_a}{\mathrm{D}^2}, \quad (3)$$

$$\frac{d\mathrm{M}}{d(-\eta_b)}=a\eta_a(1+e_a)c_ae_b\frac{(1+\eta_a)e_b+(1+e_b)\eta_a}{\mathrm{D}^2}, \quad (4)$$

$$\frac{d\mathrm{N}}{de_a}=a(1+\eta_a)(1+\eta_b)e_b\eta_b\frac{(1+\eta_a)e_b+(1+e_b)\eta_a}{\mathrm{D}^2}, \quad (5)$$

$$\frac{dN}{de_a} = a(1+\eta_a)(1+e_a)\eta_a\eta_b\frac{(1+\eta_b)e_a+(1+e_a)\eta_b}{D^2}, \quad . \quad (6)$$

$$\frac{dN}{d(-\eta_a)} = a(1+e_a)(1+e_b)\eta_b e_b\frac{(1+\eta_b)e_a+(1+e_a)\eta_b}{D^2}, \quad . \quad (7)$$

$$\frac{dN}{d(-\eta_b)} = a(1+\eta_a)(1+e_a)e_a e_b\frac{(1+\eta)e_a e_b+(1+e_b)\eta_a}{D^2}. \quad . \quad (8)$$

The implications of this analysis are summed up in the following table, where the signs of M and N and of their several differentials are set out for varying conditions of η_a and η_b.

	When $-\eta_a > 1^m$ and $-\eta_b > 1^n$	When $-\eta_a > 1$ and $-\eta_b < 1$	When $-\eta_a < 1$ and $-\eta_b > 1$	When $-\eta_a < 1$ and $-\eta_b < 1$
Sign of M . .	+	±	±	±
,, N . .	+	±	±	±
,, $\frac{dM}{de_a}$.	+	– ve	±	±
,, $\frac{dN}{de_a}$.	+	–	±	±
,, $\frac{dM}{de_b}$.	+	±	+	±
,, $\frac{dN}{de_b}$.	+	±	–	±
,, $\frac{dM}{d(-\eta_a)}$.	+	±	–	±
,, $\frac{dN}{d(-\eta_a)}$.	+	±	+	±
,, $\frac{dM}{d(-\eta_b)}$.	+	+	±	±
,, $\frac{dN}{d(-\eta_b)}$.	+	+	±	±

In the special case where $e_a = e_b = e$, and $\eta_a = \eta_b = \eta$, we have seen that

$$M = -a\frac{e\eta}{e-\eta}$$

and

$$N = -a\frac{e(1+\eta)}{e-\eta}.$$

Since e is positive and η negative, it is obvious that M is always positive; while N is positive or negative according as $-\eta >$ or < 1. Further,

$$\frac{dM}{de} = a\left(\frac{\eta}{e-\eta}\right)^2 \text{ : which is necessarily positive.}$$

$$\frac{dM}{d(-\eta)} = a\frac{e(e+2\eta)}{(e-\eta)^2} \text{ : which may be either positive or negative.}$$

$$\frac{dN}{de} = a\frac{\eta(1+\eta)}{(e-\eta)^2} \text{ : which is positive or negative according as } -\eta > \text{ or } < 1.$$

$$\frac{dN}{d(-\eta)} = a\frac{e(1+e)}{(e-\eta)^2} \text{ : which is necessarily positive.}$$

Problem 4.—To determine the comparative effects on the aggregate quantity of work done by Robinson and Jones together (1) of a (small) equiproportional increase in Jones's desire for commodity B, and (2) of a (small) equiproportional improvement in productivity in respect of commodity A, in an economy of the kind examined in the preceding sections, as against an economy where each producer consumes the whole of his own output.

An economy where each producer consumes exclusively his own product is a Crusoe economy as described in Appendix III. Write M′ and N′ for the effect on the aggregate quantity of work done in that economy of (1) a (small) equiproportional increase in desire for commodity A—now consumed by Robinson himself—and (2) a (small) equiproportional improvement in productivity in respect of commodity A.

Then, from the Problems 1 and 2 of Appendix III.,

$$M' = -a\frac{e_a\eta_a}{e_a-\eta_b}$$

$$N' = -a\frac{e_a(1+\eta_a)}{e_a-\eta_a}.$$

It follows that

$$\frac{M}{M'} = (e_a-\eta_a)\frac{e_b}{e_a}\cdot\frac{\{(1+\eta_b)e_a+(1+e_a)\eta_b\}}{\{(1+\eta_a)(1+\eta_b)e_ab_a-(1+e_a)(1+e_b)\eta_a\eta_b\}},$$

$$\frac{N}{N'} = (e_a-\eta_b)\frac{e_b}{e_a}\cdot\frac{\{(1+\eta_b)e_a+(1+e_a)\eta_b\}}{\{(1+\eta_a)(1+\eta_b)e_ae_b-(1+e_a)(1+e_b)\eta_a\eta_b\}}.$$

In the special case where $e_a = e_b$ and $\eta_a = \eta_b$, these equations give

$$M' = M,$$
$$N' = N.$$

Problem 5.—There is a chain of n Robinsons A, B, . . . Y, Z,

all of whom initially do equal quantities of work. Each member of the chain except the last buys for consumption the whole product of his successor with his own whole product; and the last member buys the whole product of the first. To determine the way in which the effect on the aggregate quantity of work done by all the Robinsons (1) of a (small) equiproportional increase in Z's desire for the produce of A, and (2) of a (small) equiproportional improvement in Z's productivity, is related to the size of n.

Consider first an increase in Z's desire for the product of A. Let $a, b, \ldots z$ be the quantities of work done by the several Robinsons and $a, b, \ldots z$ the quantities of product. As in the earlier problems let units be so chosen that in the initial situation $a = b = \ldots = z$. Let $f_a(a), f_b(b) \ldots f_y(y), f_z(z)$ be the marginal aversions from work: and $\phi_a(b), \ldots \phi_y(z)$ and $c\phi_z(a)$ the marginal desire of each Robinson for the product he buys.

Then we have a chain of equations

$$f_a(a) = \frac{b}{a}\phi_a(b), \qquad \ldots \qquad (1)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$f_y(y) = \frac{z}{y}\phi_y(z), \qquad \ldots \qquad (2)$$

$$f_z(z) = c\frac{a}{z}\phi_z(a). \qquad \ldots \qquad (3)$$

Differentiating to c (which, as before, is initially put $= 1$), we obtain a second chain

$$\frac{d}{dc}f_a(a) = \frac{d}{dc}\left\{\frac{b}{a}\phi_a(b)\right\}, \qquad \ldots \qquad (1)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\frac{d}{dc}f_y(y) = \frac{d}{dc}\left\{\frac{z}{y}\cdot\phi_y(z)\right\}, \qquad \ldots \qquad (2)$$

$$\frac{d}{dc}f_z(z) = \frac{d}{dc}\left\{c\frac{a}{z}\phi_z(a)\right\}. \qquad \ldots \qquad (3)$$

As in the earlier problems, put $\frac{1}{e_a} = \frac{af_a'(a)}{f_a(a)}$; and so on,

$\frac{1}{\eta_b} = \frac{a\phi_a'(b)}{\phi_a(b)}$; and so on.

This implies, of course, $\frac{1}{\eta_a} = \frac{z\phi_z'(a)}{\phi_z(a)}$.

Since $a=b=\ldots=z$, we then find from the above equations

$$\frac{da}{dc}\left(1+\frac{1}{e_a}\right)=\frac{db}{dc}\left(1+\frac{1}{\eta_b}\right),$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\frac{dy}{dc}\left(1+\frac{1}{e_y}\right)=\frac{dz}{dc}\left(1+\frac{1}{\eta_z}\right),$$

$$\frac{dz}{dc}\left(1+\frac{1}{e_z}\right)=\frac{da}{dc}\left(1+\frac{1}{\eta_a}\right)+a.$$

From these equations we can determine the value of $\left(\frac{da}{dc}+\ldots\frac{dz}{dc}\right)$ in terms of a and all the e's and η's. In the general case the expression for this value is highly complex. But in the special case where $e_a=e_b=\ldots=e_z$, and $\eta_a=\eta_b=\ldots=\eta_z$, if m be written for $\frac{(1+\eta)e}{(1+e)\eta}$, we find, for a chain containing n members,

$$\frac{dz}{dc}=\frac{da}{dc}m+a\frac{\eta}{1+\eta}m.$$

But
$$\frac{dz}{dc}=\frac{da}{dc}\frac{1}{m^{n-1}}$$

$$\therefore\ \frac{dz}{dc}=\frac{m}{1-m^n}\cdot a\frac{\eta}{1+\eta}.$$

Also
$$\left\{\frac{da}{dc}+\frac{db}{dc}+\ldots\frac{dz}{dc}\right\}=\frac{m}{1-m^n}a\frac{\eta}{1+\eta}\{1+\ldots m^{n-1}\}$$

$$=\frac{m}{1-m}a\frac{\eta}{1+\eta}$$

$$=-a\frac{e\eta}{e-\eta}.$$

That is to say,

$$\left\{\frac{da}{dc}+\frac{db}{dc}+\ldots\frac{dz}{dc}\right\}=-a\frac{e\eta}{e-\eta},$$

whatever be the value of n.

By a precisely analogous argument it can be shown that, in the case of an equiproportional improvement in Z's productivity,

$$\left\{\frac{da}{dc}+\frac{db}{dc}+\ldots\frac{dz}{dc}\right\}=-a\frac{e(1+\eta)}{e-\eta},$$

whatever be the value of n.

Problem 6.—In a community consisting of Robinson and

Jones only, let them no longer confine themselves wholly to each other's products, but consume also some of their own. To determine in this situation the effect on the aggregate quantity of work done by Robinson and Jones together, (1) of a (small) equiproportional increase in *total* desire for Jones's product B, and (2) of a (small) equiproportional improvement in productivity in respect of commodity A.

Let a be the quantity of Robinson's work and output: b the quantity of Jones's: α Robinson's consumption of his own commodity and β Jones's consumption of his: where $a=b$ and $\alpha=\beta$.

In the case of an increase in total desire for B, let $f_a(a)$ and $f_b(b)$ be Robinson's and Jones's marginal aversions from work; $\psi_a(\alpha)$ Robinson's marginal desire for α of A; $\phi_a(a-\alpha)$ Jones's marginal desire for $(a-\alpha)$ of A; $c\phi_b(b-\beta)$ Robinson's marginal desire for $(b-\beta)$ of B; and $c\psi_b(\beta)$ Jones's marginal desire for β of B.

We then have the four following equations:

$$f_a(a)=\psi_a(\alpha), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$f_a(a)=c\frac{b-\beta}{a-\alpha}\,.\,\phi_b(b-\beta), \quad . \quad . \quad . \quad (2)$$

$$f_b(b)=\frac{a-\alpha}{b-\beta}\phi_a(a-\alpha), \quad . \quad . \quad . \quad (3)$$

$$f_b(b)=c\psi_b(\beta). \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We have also four further equations, obtained in the way explained in the preceding sections, by differentiating each of the above elements to c.

In the case of an improvement of productivity in respect of A the relevant equations are found, on the plan of Problem 2, to be

$$f_a(a)=c\psi_a(c\alpha), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$f_a(a)=\frac{b-\beta}{a-\alpha}\phi_b(b-\beta), \quad . \quad . \quad . \quad . \quad (2)$$

$$f_b(b)=c\frac{(a-\alpha)}{b-\beta}\phi_a\{c(a-\alpha)\}, \quad . \quad . \quad . \quad (3)$$

$$f_b(b)=\psi_b(\beta), \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

together with four further equations, obtained by differentiating each of the elements to c.

In the general case for both these problems the formulae obtained from the equations for $\left(\frac{da}{dc}+\frac{db}{dc}\right)$ are too complex to be serviceable. But in the special case where all the elasticities of marginal desire for commodity are the same and both the elasticities of marginal aversion from work are the same, they

reduce to simple forms. Write e for elasticity of marginal aversion from work and η for elasticity of marginal desire for commodity. These elasticities are then defined, in the manner of p. 285, thus:

$$\frac{1}{e}=\frac{af_a'(a)}{f_a(a)}=\frac{bf_b'(b)}{f_b(b)},$$

$$\frac{1}{\eta}=\frac{\alpha\psi_a'(\alpha)}{\psi_a(\alpha)}=\frac{(a-\alpha)\phi_a'(a-\alpha)}{\phi_a(a-\alpha)}$$

$$=\frac{\beta\psi_b'(\beta)}{\psi_b(\beta)}=\frac{(b-\beta)\phi_b'(b-\beta)}{\phi_b(b-\beta)}.$$

Manipulating these various equalities we find that, in the case of an increase in total desire for A,

$$\left(\frac{da}{dc}+\frac{db}{dc}\right)=-a\frac{e\eta}{e-\eta}:$$

in the case of an improvement in productivity in respect of A

$$\left(\frac{da}{dc}+\frac{db}{dc}\right)=-a\frac{e(1-\eta)}{e-\eta}.$$

These equalities hold whatever the values of α/a and β/b—values which, since $a=b$ and $\alpha=\beta$, are, of course, identical.

Problem 7.—In an assembly of n Robinsons each making a single commodity and exchanging a part of his output with each of the others, to determine the effect on the aggregate quantity of work done by all of them (1) of an equiproportional increase in total desire for one commodity; (2) of an equiproportional improvement in productivity in respect of one commodity.

Here again the general cases yield hopelessly complex formulae. In the special case of a perfectly symmetrical system, as defined in Chapter XXI. § 8, *i.e.* where in the initial situation everybody is consuming the same quantity of everything, it appears that yet again, with an equiproportional increase in total desire for any one commodity

$$\frac{da}{dc}+\frac{db}{dc}+\ .\ .\ .\ =-a\frac{e\eta}{e-\eta},$$

and with an equiproportional improvement of productivity in respect of one commodity

$$\frac{da}{dc}+\frac{db}{dc}+\ .\ .\ .\ =-a\frac{e(1-\eta)}{e-\eta}.$$

These equalities hold whatever the value of n.

VIII

Appendix to Chapter XXXI

The relation between price and quantity of work offered by a representative worker.

§ 1. Assume that the representative worker's desire attitude to payment is independent of the total quantity of work that he is doing, and his aversion attitude from work to the total quantity of payment he is receiving.

Write x for quantity of work, p for price per unit of it, $f(x)$ for his marginal aversion from work and $\phi(px)$ for his marginal desire for payment.

Also write E for the elasticity of his supply of work in response to price, e for the elasticity of his marginal aversion from work, and η for the elasticity of his marginal desire for payment. We then have

$$\frac{1}{\mathrm{E}} = \frac{dp}{dx} \cdot \frac{x}{p}.$$

But

$$p = \frac{f(x)}{\phi(px)}.$$

$$\therefore \frac{1}{\mathrm{E}} = \frac{d}{dx}\left\{\frac{f(x)}{\phi(px)}\right\} \cdot x\frac{\phi(px)}{f(x)}$$

$$= \frac{xf'}{f} - \frac{px}{\phi} \cdot \frac{d\phi}{d(px)}\left\{1 + \frac{dp}{dx} \cdot \frac{x}{p}\right\}.$$

But

$$\frac{1}{e} = \frac{xf'}{f}$$

and

$$\frac{1}{\eta} = \frac{px}{\phi} \cdot \frac{d\phi}{d(px)}.$$

$$\therefore \frac{1}{\mathrm{E}} = \frac{1}{e} - \frac{1}{\eta}\left(1 + \frac{1}{\mathrm{E}}\right).$$

$$\therefore \mathrm{E} = \frac{1 + \frac{1}{\eta}}{\frac{1}{e} - \frac{1}{\eta}} = -\frac{e(1+\eta)}{e-\eta}. \qquad \text{(I)}$$

Since e is obviously positive and η negative, the denominator of this expression must be positive. It follows that E is positive or negative according as η is numerically $>$ or < 1.

§ 2. Let us now allow our representative worker's aversion attitude from work to depend in part on the total quantity of pay that he is receiving, so that now

$$p = \frac{f\{x, px\}}{\phi(px)}.$$

Therefore, in place of xf'/f in the above analysis, we have

$$\frac{x}{f} \cdot \frac{df(x, px)}{dx},$$

$$= \frac{x}{f}\left\{\frac{\partial f}{\partial x} + \frac{d(px)}{dx} \frac{\partial f}{\partial(px)}\right\}$$

$$= \frac{x}{f}\frac{\partial f}{\partial x} + \frac{px}{f} \frac{\partial f}{\partial(px)}\left\{1 + \frac{x}{p}\frac{dp}{dx}\right\}.$$

Now let e mean the elasticity of Robinson's marginal aversion from work on the assumption that his aggregate payment is constant, and ϵ the elasticity of his marginal aversion from work *in respect of changes in his aggregate payment*, namely, a (small) proportionate change in this marginal aversion divided into the associated proportionate change in his aggregate payment.

Then
$$\frac{1}{e} = x\frac{\partial f}{dx} \div f,$$

$$\frac{1}{\epsilon} = px\frac{\partial f}{\partial(px)} \div f.$$

$$\therefore \ \frac{1}{\mathrm{E}} = \frac{1}{e} + \frac{1}{\epsilon}\left(1 + \frac{1}{\mathrm{E}}\right) - \frac{1}{\eta}\left(1 + \frac{1}{\mathrm{E}}\right).$$

$$\therefore \ \mathrm{E} = \frac{1 + \frac{1}{\eta} - \frac{1}{\epsilon}}{\frac{1}{e} - \frac{1}{\eta} + \frac{1}{\epsilon}} = -\frac{e(1+\eta) - \frac{e\eta}{\epsilon}}{e - \eta - \frac{e\eta}{\epsilon}}. \qquad \text{(II)}$$

Now ϵ is clearly positive. Therefore the denominator of this expression is positive. It follows that E is positive or negative according as η is numerically $>$ or $<$ unity *plus* some assigned quantity. The assigned quantity is larger, and E is more likely to be negative, the smaller is ϵ.

§ 3. If E is positive, it is obvious that an increase in the price offered per unit of work must cause our representative worker to

perform more units, thereby, of course, securing a bigger aggregate of pay. But, if, over the relevant range, E is negative, we must not, as we might be inclined to do at first sight, assert forthwith, as an *a priori* truth, that the converse of this proposition holds. For the relevant part of a supply curve that is negatively inclined may have either of two *directions*. If we imagine a particle moving along the curve from its starting-point on the vertical axis, this particle, on the negatively inclined part of the curve, may be either passing away from or approaching towards that axis. The curve in its relevant part may then, to use Mr. Kahn's language,[1] be either forward-falling or backward-rising. Its direction does not make any difference to the sign or magnitude of its elasticity at any point, but it makes a great difference to the consequences that follow from a rise in the demand curve for work. At a later stage in another connection this point will be found to be important.[2] It is, however, plainly out of the question that the supply curve of work offered for pay by a single representative worker, or any constant number of such workers, of given capacity, should be forward-falling. For this would mean that they were ready to perform larger totals of work for lower rates of pay. This is impossible even if our representative worker's attitude to work is independent of the amount of his income: if it is not independent, but larger incomes are associated with greater aversion, it is still more impossible. Hence, so far as the present problem is concerned, if the representative worker's supply curve is negatively inclined, it must be backward-rising. Now, while with either a forward-falling or a forward-rising supply curve, equilibrium will only be stable if the demand curve lies above the supply curve to the left of the point of intersection, *i.e.* if the elasticity of demand is numerically less than that of supply, with a backward-rising supply curve equilibrium will only be stable if the demand curve lies above the supply curve to the right of the intersection point, *i.e.* if the elasticity of demand is numerically greater than the elasticity of supply. Any reader who will trouble to draw a diagram—remembering that the demand curve must be negatively inclined—can readily satisfy himself on this matter. Since unstable positions of equilibrium cannot in practice be maintained, they may be left out of account, at all events in a study of stationary states. Hence, when, in the face of a backward-rising supply curve, the demand curve rises, provided there is stability at all, in the new situation, while price is higher, the quantity of the commodity or service that

[1] Cf. Note on elasticity of substitution, *Review of Economic Studies*, Oct. 1933.

[2] Cf. *post*, Appendix XI. § 12.

is sold will be smaller than it was before. Though, therefore, it is not an obvious *a priori* truth, it is, nevertheless, the fact that, if E has a negative value, a rise in the price offered for work must cause the quantity of work provided by a representative worker of given capacity to decrease.

IX

Appendix to Chapter XXXII

The relation between the proportions of their income that people with large and small incomes will save at a given rate of interest if their rates of discounting future satisfactions are the same.

In order that either class of person may make any net saving it is necessary, as has been shown in the text, that the rate of interest should exceed the rate of discounting future satisfactions. Let then the former be i per unit per annum: the latter q per unit. Suppose, for simplicity—this does not affect the substance of the argument—that savings can be made for one year only. Write x for quantity of income; $f(x)$ for aggregate quantity of satisfaction derived from the consumption of x units of income and a for the amount saved from this year's income. It is postulated that, apart from saving, everybody's next year's income will be the same as this year's. Then there is available for consumption this year $(x-a)$ units, next year $\{x+(1+i)a\}$ units. It is necessary, therefore, for equilibrium that

$$\frac{d}{dx}\left\{f(x-a)\right\}=\frac{1}{(1+q)}\cdot\frac{d}{dx}\Big[f\{x+(1+i)a\}\Big].$$

$\therefore$ as a first approximation—an exact approximation if the curve connecting amount of income and its marginal utility is linear—

$$(1+q)(f'-af'')=f'+(1+i)af'',$$
$$\therefore\ qf'=(2+i+q)af''.$$

Write η for the elasticity of the income-marginal-utility curve in respect of income x.

Then $$\eta=\frac{f'(x)}{xf''(x)}.$$

$$\therefore\qquad \frac{a}{x}=\frac{q}{2+i+q}\cdot\frac{f'}{xf''}=\frac{q}{2+i+q}\eta.$$

It follows that a/x varies directly with η. But, as is shown in the text of Chapter XXXII, η is likely to be larger in respect of large incomes than in respect of small. Therefore a larger proportion of large incomes than of small will be saved, even though the holders of the two sizes of income discount future satisfactions at the same rate.

X

Appendix to Chapter XXXIII

The general condition of equilibrium in a one-commodity community where the productivity function is homogeneous in the first degree.

Write x, y, z for quantities of the several factors of production X, Y, Z, and P for the total product. There is then a productivity function F, such that $F(x, y, z) = P$. Since this function is homogeneous in the first degree, the marginal private and marginal collective products of each factor are identical, and are equal respectively to $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$ and $\frac{\partial F}{\partial z}$.

Write $\frac{\partial\psi(x, y, z)}{\partial x}$, $\frac{\partial\psi(x, y, z)}{\partial y}$ and $\frac{\partial\psi(x, y, z)}{\partial z}$ for the maintenance prices of the several factors. Then for the system to be in equilibrium, provided that there is no monopoly anywhere, the following equations must be satisfied.

$$\frac{\partial F}{\partial x} = \frac{\partial\psi}{\partial x}, \qquad \text{(I)}$$

$$\frac{\partial F}{\partial y} = \frac{\partial\psi}{\partial y}, \qquad \text{(II)}$$

$$\frac{\partial F}{\partial z} = \frac{\partial\psi}{\partial z}. \qquad \text{(III)}$$

If monopoly is exercised in respect of the whole of one factor only, the quantity of this factor being represented by z, the corresponding equations are

$$\frac{\partial F}{\partial x} = \frac{\partial\psi}{\partial x}, \qquad \text{(I)}$$

$$\frac{\partial F}{\partial y} = \frac{\partial\psi}{\partial y}, \qquad \text{(II)}$$

$$\frac{\partial F}{\partial z}+z\frac{\partial^2 F}{\partial z^2}=\frac{\partial \psi}{\partial z}. \quad . \quad . \quad . \quad \text{(III)}$$

In both cases equally the number of independent equations is equal to the number of unknowns, so that the quantities of the several factors are mathematically determinate. Of course, when monopoly is present, the values of *all* the unknowns, not merely of that referring to the monopolistic factor, are different from what they are when there is no monopoly.

XI

APPENDIX TO CHAPTER XXXIV

I

The conditions in which in a one-commodity community, with a homogeneous function, an increase in the quantity of one factor entails an increase in the absolute remuneration of that factor.

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§ 1. SUPPOSE that a shift in the maintenance conditions affecting X has taken place, in such wise that, when everything has settled down in the new stationary state, the quantity of X is $(x+\Delta x)$ instead of x, but all the elasticities of partial marginal productivity and all the elasticities of maintenance are the same as they were before. Let the quantities of the three factors in the initial equilibrium be x, y and z; and the productivity function $F(x, y, z)$. Write E_x for the elasticity of total marginal productivity of X; ${}_x\epsilon_x$ for the elasticity of partial marginal productivity of X in respect of changes in x; ${}_y\epsilon_x$ for the elasticity of partial marginal productivity of X in respect of changes in y; and so on. Write also ${}_xe_x$, ${}_xe_y$ and ${}_xe_z$ for the elasticities of maintenance of the three factors in respect of changes in x; and corresponding expressions for their elasticities of maintenance in respect of changes in y and z. Further, write p_x for $\frac{\partial F(x, y, z)}{\partial x}$, namely, the marginal productivity of x units of X, and p_y, p_z with analogous meanings. Finally, choose units of quantity in such a way that x, y and z are all equal.

§ 2. When the quantity of X increases, the rate of change in

$$\begin{aligned}\text{X's total remuneration } &= \frac{d}{dx}\left\{x\frac{\partial F(x, y, z)}{\partial x}\right\}\\ &= \frac{\partial F}{\partial x} + x\frac{d}{dx}\frac{\partial F}{\partial x}\\ &= p_x\left(1 + \frac{1}{E_x}\right).\end{aligned}$$

This is positive or negative according as, on the one hand, E_x is positive, or, being negative, is numerically greater than unity,

or, on the other hand, is negative and numerically less than unity. Our problem, therefore, is to determine in what condition E_x is in fact positive, or, being negative, is numerically greater than unity.

§ 3. The values of the several ϵ's may be set out as follows:

$$\frac{1}{{}_x\epsilon_x}=x\frac{\partial}{\partial x}\left\{\frac{\partial F}{\partial x}\right\}\div\frac{\partial F}{\partial x}=\frac{x}{p_x}\cdot\frac{\partial}{\partial x}\left\{\frac{\partial F}{\partial x}\right\},$$

$$\frac{1}{{}_x\epsilon_y}=\frac{x}{p_y}\frac{\partial}{\partial x}\left\{\frac{\partial F}{\partial y}\right\},$$

$$\frac{1}{{}_y\epsilon_y}=\frac{y}{p_y}\frac{\partial}{\partial y}\left\{\frac{\partial F}{\partial y}\right\},$$

$$\frac{1}{{}_y\epsilon_x}=\frac{y}{p_x}\frac{\partial}{\partial y}\left\{\frac{\partial F}{\partial x}\right\},$$

and so on.

§ 4. By definition: $\frac{1}{E_x}=\frac{x}{p_x}\cdot\frac{d}{dx}(p_x)$

$$=\frac{x}{p_x}\left\{\frac{p_x}{x}\cdot\frac{1}{{}_x\epsilon_x}+\frac{dy}{dx}\cdot\frac{p_x}{y}\cdot\frac{1}{{}_y\epsilon_x}+\frac{dz}{dx}\cdot\frac{p_x}{z}\cdot\frac{1}{{}_z\epsilon_x}\right\}$$

$$=\frac{1}{{}_x\epsilon_x}+\frac{dy}{dx}\cdot\frac{1}{{}_y\epsilon_x}+\frac{dz}{dx}\cdot\frac{1}{{}_z\epsilon_x}. \qquad \text{(I)}$$

The values of dy/dx and dz/dx are determined by the condition that, in the new equilibrium, the maintenance price of any factor differs from the original maintenance price to the same extent that the new marginal product of the factor differs from the old one. Hence, the maintenance price of y units of Y being written $f_y(x, y, z)$, and that of z units of Z being written $f_z(x, y, z)$, it follows that

$$\frac{d}{dx}\frac{\partial F}{\partial y}=\frac{d}{dx}f_y=\frac{\partial f_y}{\partial x}+\frac{dy}{dx}\frac{\partial f_y}{\partial y}+\frac{dz}{dx}\frac{\partial f_y}{\partial z}$$

and

$$\frac{d}{dx}\frac{\partial F}{\partial z}=\frac{d}{dx}f_z=\frac{\partial f_z}{\partial x}+\frac{dy}{dx}\frac{\partial f_z}{\partial y}+\frac{dz}{dx}\frac{\partial f_z}{\partial z}.$$

By an easy transformation these equations become

$$\left\{\frac{1}{{}_x\epsilon_y}-\frac{1}{{}_xe_y}\right\}+\frac{dy}{dx}\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}+\frac{dz}{dx}\left\{\frac{1}{{}_z\epsilon_y}-\frac{1}{{}_ze_y}\right\}=0 \qquad \text{(II)}$$

and

$$\left\{\frac{1}{{}_x\epsilon_z}-\frac{1}{{}_xe_z}\right\}+\frac{dy}{dx}\left\{\frac{1}{{}_y\epsilon_z}-\frac{1}{{}_ye_z}\right\}+\frac{dz}{dx}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}=0. \qquad \text{(III)}$$

If the several factors have completely independent maintenance functions, these equations are simplified to the extent that

$\frac{1}{{}_x e_y}$, $\frac{1}{{}_z e_y}$, $\frac{1}{{}_x e_z}$ and $\frac{1}{{}_y e_z}$ are all nil. There is, however, no difficulty in solving them in the more general case. In that case

$$\frac{dy}{dx}=\frac{\left\{\frac{1}{{}_z\epsilon_y}-\frac{1}{{}_ze_y}\right\}\left\{\frac{1}{{}_x\epsilon_z}-\frac{1}{{}_xe_z}\right\}-\left\{\frac{1}{{}_x\epsilon_y}-\frac{1}{{}_xe_y}\right\}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}}{\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}-\left\{\frac{1}{{}_z\epsilon_y}-\frac{1}{{}_ze_y}\right\}\left\{\frac{1}{{}_y\epsilon_z}-\frac{1}{{}_ye_z}\right\}}, \quad . \quad \text{(IV)}$$

$$\frac{dz}{dx}=\frac{\left\{\frac{1}{{}_x\epsilon_y}-\frac{1}{{}_xe_y}\right\}\left\{\frac{1}{{}_y\epsilon_z}-\frac{1}{{}_ye_z}\right\}-\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}\left\{\frac{1}{{}_x\epsilon_z}-\frac{1}{{}_xe_z}\right\}}{\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}-\left\{\frac{1}{{}_z\epsilon_y}-\frac{1}{{}_ze_y}\right\}\left\{\frac{1}{{}_y\epsilon_z}-\frac{1}{{}_ye_z}\right\}}. \quad . \quad \text{(V)}$$

When the maintenance functions of the several factors are independent, these expressions reduce to

$$\frac{dy}{dx}=\frac{\frac{1}{{}_z\epsilon_y}\cdot\frac{1}{{}_x\epsilon_z}-\frac{1}{{}_x\epsilon_y}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}}{\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}-\frac{1}{{}_z\epsilon_y}\cdot\frac{1}{{}_y\epsilon_z}}, \quad . \quad . \quad \text{(VI)}$$

$$\frac{dz}{dx}=\frac{\frac{1}{{}_x\epsilon_y}\cdot\frac{1}{{}_y\epsilon_z}-\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}\frac{1}{{}_x\epsilon_z}}{\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}\left\{\frac{1}{{}_z\epsilon_z}-\frac{1}{{}_ze_z}\right\}-\frac{1}{{}_z\epsilon_y}\cdot\frac{1}{{}_y\epsilon_z}}. \quad . \quad . \quad \text{(VII)}$$

By writing the values for dy/dx and dz/dx into equation (I) we obtain the value of E_x in terms of nine ϵ's and six e's, *i.e.* all the e's excepting those affecting the factor X, for the general case; and in terms of nine ϵ's and two e's for the case in which the maintenance conditions of the several factors are independent.

§ 5. In the special case of only two factors of production all the elements involving z disappear.

Hence from (IV)

$$\frac{dy}{dx}=-\frac{\left\{\frac{1}{{}_x\epsilon_y}-\frac{1}{{}_xe_y}\right\}}{\left\{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}\right\}}. \quad . \quad . \quad . \quad \text{(VIII)}$$

But from (I)

$$\frac{1}{E_x}=\frac{1}{{}_x\epsilon_x}+\frac{dy}{dx}\cdot\frac{1}{{}_y\epsilon_x}.$$

Hence

$$\frac{1}{E_x}=\frac{1}{{}_x\epsilon_x}-\frac{1}{{}_y\epsilon_x}\cdot\frac{\frac{1}{{}_x\epsilon_y}-\frac{1}{{}_xe_y}}{\frac{1}{{}_y\epsilon_y}-\frac{1}{{}_ye_y}}. \quad . \quad . \quad \text{(IX)}$$

§ 6. Since the productivity function is homogeneous in the first degree, an equal proportionate change in the quantities of all the factors must leave the marginal productivity of each factor unaltered. Hence, when units are so chosen that $x=y=z$ it is necessary that

$$\frac{\partial}{\partial x}p_x+\frac{\partial}{\partial y}p_x+\frac{\partial}{\partial z}p_x=0,$$

$$\frac{\partial}{\partial x}p_y+\frac{\partial}{\partial y}p_y+\frac{\partial}{\partial z}p_y=0,$$

$$\frac{\partial}{\partial x}p_z+\frac{\partial}{\partial y}p_z+\frac{\partial}{\partial y}p_z=0.$$

Hence

$$\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_x}+\frac{1}{{}_z\epsilon_x}=0, \quad . \quad . \quad . \quad \text{(X)}$$

$$\frac{1}{{}_x\epsilon_y}+\frac{1}{{}_y\epsilon_y}+\frac{1}{{}_z\epsilon_y}=0, \quad . \quad . \quad . \quad \text{(XI)}$$

$$\frac{1}{{}_x\epsilon_z}+\frac{1}{{}_y\epsilon_z}+\frac{1}{{}_z\epsilon_z}=0. \quad . \quad . \quad . \quad \text{(XII)}$$

When there are only two factors of production, these equations reduce to

$$\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_x}=0, \quad . \quad . \quad . \quad . \quad \text{(XIII)}$$

$$\frac{1}{{}_x\epsilon_y}+\frac{1}{{}_y\epsilon_y}=0. \quad . \quad . \quad . \quad . \quad \text{(XIV)}$$

§ 7. It follows from (IX) and (XIII) that, with two factors of production, when maintenance functions are not independent,

$$\frac{1}{\mathrm{E}_x}=\frac{1}{{}_x\epsilon_x}\left\{1-\frac{\dfrac{1}{{}_y\epsilon_y}+\dfrac{1}{{}_xe_y}}{\dfrac{1}{{}_y\epsilon_y}-\dfrac{1}{{}_ye_y}}\right\}$$

$$=\frac{1}{{}_x\epsilon_x}\,\frac{{}_y\epsilon_y\left\{1+\dfrac{{}_ye_y}{{}_xe_y}\right\}}{{}_y\epsilon_y-{}_ye_y} \quad . \quad . \quad . \quad \text{(XV)}$$

Hence, when the maintenance conditions of X and Y are independent

$$\frac{1}{\mathrm{E}_x}=\frac{1}{{}_x\epsilon_x}\cdot\frac{{}_y\epsilon_y}{{}_y\epsilon_y-{}_ye_y} \quad . \quad . \quad . \quad \text{(XVI)}$$

§ 8. From the two last formulae certain inferences can be derived as to the conditions in which E_x is likely to be either

positive, or, if negative, numerically less than unity. When the maintenance conditions of both X and Y are independent, we find from equation (XVI) that, with ${}_ye_y$ positive, E_x is necessarily negative; and that it is a larger negative quantity—and so more likely to be numerically > 1—(1) the larger numerically is ${}_x\epsilon_x$, (2) the smaller numerically is ${}_y\epsilon_y$, and (3) the larger is ${}_ye_y$. If ${}_ye_y$ is negatived, the situation is, however, less clear. In Appendix VIII. a distinction was drawn between two forms of maintenance curve, for both of which the elasticity is negative, namely, (*a*) forward-falling curves, like Marshall's increasing return curves, and (*b*) backward-rising curves. With curves of the former type it was noted that in stable equilibrium the demand curve must lie above the maintenance curve to the left of their point of intersection. This implies that ${}_ye_y$ is a larger negative quantity than ${}_y\epsilon_y$. Stationary states involving unstable positions of equilibrium may be ruled out of account. We may, therefore, assume that, if ${}_ye_y$, is negative, $({}_y\epsilon_y - {}_ye_y)$ must be positive. This implies that E_x is positive. With curves of the backward-rising type, stable equilibrium requires that to the left of their point of intersection the demand curve shall lie below the maintenance curve, *i.e.* that ${}_ye_y$ shall have a smaller negative value than ${}_y\epsilon_y$. Hence a negative value of ${}_ye_y$ implies that $({}_y\epsilon_y - {}_ye_y)$ must be negative. In this situation E_x must be negative: and its negative value is greater, and so more likely to be numerically > 1, (1) the larger numerically is ${}_x\epsilon_x$, (2) the larger numerically is ${}_y\epsilon_y$, and (3) the smaller numerically is ${}_ye_y$.

§ 9. When the maintenance conditions of X and Y are interdependent, equation (XV) becomes relevant instead of equation (XVI). If ${}_ye_y$ is positive, it is obvious that the existence of ${}_xe_y$ makes E_x respectively a larger or a smaller negative quantity than it would otherwise have been, according as ${}_xe_y$ is negative or positive: and the effect in either direction is larger the smaller numerically is ${}_xe_y$. If ${}_ye_y$ is negative and the maintenance curve forward-falling, E_x must be positive unless ${}_xe_y$ is both positive and numerically smaller than ${}_ye_y$; in which case it will be negative. If ${}_ye_y$ is negative and the curve backward-rising, E_x will be positive for negative values of ${}_xe_y$ and for positive values numerically smaller than ${}_ye_y$. For positive values of ${}_xe_y$ numerically larger than ${}_ye_y$, E_x will be negative, and its numerical value will be larger, the smaller is ${}_xe_y$ if ${}_xe_y$ is positive, and the larger if it is negative.[1]

[1] It would be much more convenient in verbal translations of these discussions to speak of the reciprocals of elasticities, which are sometimes called flexibilities, rather than of elasticities themselves. Thus an increase in the quantity of any factor entails an increase or decrease in its absolute remuneration according as its flexibility is, on the one hand, positive or, being nega-

II

The conditions in which, with a homogeneous function, an increase in the quantity of one factor entails an increase in the absolute remuneration of one of the other factors.

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§ 10. As before let there be three factors X, Y and Z, whose quantities are x, y and z. The condition for Y's aggregate remuneration being increased in consequence of an increase of X is that $\frac{d}{dx}(yp_y)$ shall be positive.

Now
$$\frac{d}{dx}(yp_y) = \frac{dy}{dx}p_y + y\frac{d}{dx}(p_y)$$
$$= p_y\frac{dy}{dx} + y\left[\frac{\partial}{\partial x}(p_y) + \frac{dy}{dx}\cdot\frac{\partial}{\partial y}(p_y) + \frac{dz}{dx}\frac{\partial}{\partial z}(p_y)\right].$$

This is positive if
$$\left\{\frac{dy}{dx} + \frac{1}{{}_x\epsilon_y} + \frac{dy}{dx}\frac{1}{{}_y\epsilon_y} + \frac{dz}{dx}\frac{1}{{}_z\epsilon_z}\right\}$$
is positive; *i.e.* if
$$\left\{\frac{1}{{}_x\epsilon_y} + \frac{dy}{dx}\left(1 + \frac{1}{{}_y\epsilon_y}\right) + \frac{dz}{dx}\frac{1}{{}_z\epsilon_z}\right\}$$
is positive.

Substituting the values for dy/dx and dz/dx given in § 3, we can, if we choose, express this condition in an exceedingly complex formula. If the maintenance conditions of X, Y and Z are all interdependent, this formula will embody all the 9 ϵ's together with all the 9 e's; if they are all independent, it will embody all the 9 ϵ's and all the 3 then relevant e's, namely ${}_xe_x$, ${}_ye_y$ and ${}_ze_z$. In neither case can any broad conclusions of a general kind be extracted from it.

§ 11. When there are only two factors of production associated in a homogeneous function of the first degree, we know from (XIV) that $\frac{1}{{}_x\epsilon_y} = -\frac{1}{{}_y\epsilon_y}$. Therefore the condition for Y's absolute remuneration to increase when the quantity of X increases is that

tive, is numerically less than unity, or, on the other hand, is negative and numerically greater than unity. In this formulation we avoid the awkwardness of having to make the next stage after a large positive elasticity an infinite negative one. The term elasticity with its customary connotation is, however, so firmly established in common usage that it has seemed best to retain it.

$$-\frac{1}{{}_y\epsilon_y}+\frac{dy}{dx}\left(1+\frac{1}{{}_y\epsilon_y}\right)$$

is positive. But, again from (VIII) and (XIV), in these conditions

$$\frac{dy}{dx}=\frac{\dfrac{1}{{}_y\epsilon_y}+\dfrac{1}{{}_xe_y}}{\dfrac{1}{{}_y\epsilon_y}-\dfrac{1}{{}_ye_y}}=\frac{{}_ye_y\left(1+\dfrac{{}_y\epsilon_y}{{}_xe_y}\right)}{{}_ye_y-{}_y\epsilon_y}.$$

Therefore, in order to Y's absolute remuneration being increased, it is necessary that

$$\frac{1+{}_ye_y+(1+{}_y\epsilon_y)\dfrac{{}_ye_y}{{}_xe_y}}{{}_ye_y-{}_y\epsilon_y}$$

shall be positive.

In the special case where the maintenance conditions of X and Y are independent, this condition reduces to the condition that $\frac{1+{}_ye_y}{{}_ye_y-{}_y\epsilon_y}$ shall be positive.

§ 12. Let us consider first the implications of this simple case. We know that ${}_y\epsilon_y$ is negative. Therefore, provided that ${}_ye_y$ is positive, this condition is necessarily satisfied. If ${}_ye_y$ is negative and the maintenance curve forward-falling, we have seen in § 8 that, for stable equilibrium, $({}_ye_y-{}_y\epsilon_y)$ must be negative. Therefore the condition will not be satisfied unless the negative value of ${}_ye_y$ is numerically less than unity. Now for ${}_ye_y$ to be numerically less than unity means that a larger total quantity of the factor Y will be maintained for a smaller total remuneration than is needed to maintain a smaller total quantity. This we may fairly hold to be impossible. It follows that in this class of case, in spite of ${}_ye_y$ being negative, the aggregate remuneration of Y is necessarily increased in consequence of an increase of X, provided that ${}_y\epsilon_y$ is not infinite: that is to say, save only in the limiting case where X and Y are perfect substitutes and the aggregate remuneration of Y is unaffected by variations in X. If ${}_ye_y$ is negative and the maintenance curve backward-rising, we have seen that, for stable equilibrium, $({}_ye_y-{}_y\epsilon_y)$ must be positive. Hence $\frac{1+{}_ye_y}{{}_ye_y-{}_y\epsilon_y}$ must be negative unless ${}_ye_y$ is numerically less than unity: a condition that we have agreed to rule out as impossible. Hence in this class of case for ${}_ye_y$ to be negative means—provided that ${}_y\epsilon_y$ is not infinite—that the aggregate remuneration of Y is diminished by an increase in X.

§ 13. In the more complex case where the maintenance conditions of X and Y are interdependent, the condition for Y's

aggregate remuneration being increased by an increase in the stock of X is, as we have seen, that

$$\frac{1 + {}_ye_y + (1 + {}_y\epsilon_y)\dfrac{{}_ye_y}{{}_xe_y}}{{}_ye_y - {}_y\epsilon_y}$$

is positive.

If ${}_ye_y$ is positive, or if it is negative and the maintenance curve is forward-falling, this condition is necessarily satisfied provided that either ${}_xe_y$ is negative and ${}_y\epsilon_y$ is numerically greater than 1, or ${}_xe_y$ is positive and ${}_y\epsilon_y$ is numerically less than 1; that is, provided that either an increase in X's quantity directly entails an increase in Y's quantity and the elasticity of demand for Y is numerically >1, or an increase in X's quantity directly entails a decrease in Y's quantity, and the elasticity of demand for Y is numerically <1. If these provisos fail, the above condition may still be satisfied, but it need not be. If ${}_ye_y$ is negative and the maintenance curve is backward-rising, the required condition cannot be satisfied when the above provisos hold: if they fail, it may be satisfied, but need not be.

III

The conditions in which, with a homogeneous function, an increase in the quantity of one factor entails an increase in the proportionate remuneration of that factor.

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§ 14. The general condition that the proportionate share of remuneration enjoyed by a factor X, whose quantity is increased, shall itself thereby be increased is, given that the productivity function is homogeneous in the first degree, that

$$\frac{d}{dx}\left\{\frac{xp_x + yp_y + zp_z}{xp_x}\right\}$$

shall be negative. Remembering that our units are so chosen as to make $x = y = z$, we obtain from this the condition that

$$\left[\left\{\frac{p_y}{p_x}\left(1 + \frac{1}{{}_x\epsilon_x} - \frac{1}{{}_x\epsilon_y}\right) + \frac{p_z}{p_x}\left(1 + \frac{1}{{}_x\epsilon_x} - \frac{1}{{}_x\epsilon_z}\right)\right\}\right.$$
$$- \frac{dy}{dx}\left\{\frac{p_y}{p_x}\left(1 + \frac{1}{{}_y\epsilon_y} - \frac{1}{{}_y\epsilon_x}\right) + \frac{p_z}{p_x}\left(\frac{1}{{}_z\epsilon_z} - \frac{1}{{}_y\epsilon_x}\right)\right\}$$
$$\left. - \frac{dz}{dx}\left\{\frac{p_y}{p_x}\left(\frac{1}{{}_z\epsilon_y} - \frac{1}{{}_z\epsilon_x}\right) + \frac{p_z}{p_x}\left(1 + \frac{1}{{}_z\epsilon_z} - \frac{1}{{}_z\epsilon_x}\right)\right\}\right]$$

shall be positive.

§ 15. If the quantities of all the factors other than X are rigidly fixed, dy/dx and dz/dx are both nil. Hence the condition for X's proportionate share being increased becomes that

$$\frac{p_y}{p_x}\left\{1+\frac{1}{{}_x\epsilon_x}-\frac{1}{{}_x\epsilon_y}\right\}+\frac{p_z}{p_x}\left\{1+\frac{1}{{}_x\epsilon_x}-\frac{1}{{}_x\epsilon_z}\right\}$$

is positive. This condition is obviously more likely to be satisfied, the larger numerically is ${}_x\epsilon_x$, and the smaller, if negative, and the larger, if positive, are ${}_x\epsilon_y$ and ${}_x\epsilon_z$.

§ 16. In the special case where there are only two factors of production, but it is not assumed that the quantity of Y is fixed, all the elements in the expression set out at the end of § 14 that contain z disappear. Moreover, when there are only two factors of production, homogeneity, as is shown in § 6, entails the equalities

$$-\frac{1}{{}_y\epsilon_x}=\frac{1}{{}_x\epsilon_x} \quad \text{and} \quad -\frac{1}{{}_x\epsilon_y}=\frac{1}{{}_y\epsilon_y}.$$

Hence the condition for the proportionate share of remuneration enjoyed by X to increase as a consequence of its quantity increasing is that

$$\left\{1-\frac{dy}{dx}\right\}\left\{1+\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right\}$$

is positive.

If the maintenance conditions of X and Y are independent, it follows from (VIII) that

$$\left(1-\frac{dy}{dx}\right)=\frac{{}_y\epsilon_y}{{}_y\epsilon_y-{}_ye_y}$$

Hence the condition for X's share being increased in consequence of an increase in its quantity is that

$$\frac{{}_y\epsilon_y}{{}_y\epsilon_y-{}_ye_y}\left\{1+\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right\}$$

is positive. We have seen in § 8 that, if ${}_ye_y$ is positive, or is negative on a backward-rising curve, $({}_y\epsilon_y-{}_ye_y)$ must be negative. Therefore, since ${}_y\epsilon_y$ is negative, the condition for X's proportionate share being increased by an increase in its quantity is that

$$\left(1+\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right)$$

shall be positive: *i.e.* that

$$\left(\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right),$$

the sum of the reciprocals of the elasticities of partial productivity of X and Y, which is of course negative, shall be numerically <1. If ${}_ye_y$ is negative on a forward-falling curve, then $({}_y\epsilon_y-{}_ye_y)$ is positive, and this condition is reversed. It is not difficult to show

that, with two factors of production with independent maintenance conditions, $\left(\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right)$ is equivalent (in respect of a homogeneous function of the first degree) to the reciprocal of what Mrs. Robinson, Dr. Hicks, Mr. Kahn and others have named the elasticity of substitution between X and Y defined as

$$\left\{\frac{\frac{d}{dx}\left\{\frac{x}{y}\right\}}{\frac{x}{y}} \div \frac{\frac{d}{dx}\left\{\frac{p_x}{p_y}\right\}}{\frac{p_x}{p_y}}\right\}.[1]$$

§ 17. It will be observed that the expression

$$\left\{1+\frac{1}{{}_x\epsilon_x}+\frac{1}{{}_y\epsilon_y}\right\}$$

is symmetrical in respect of X and Y, and, therefore, must appear also in the formula for the condition in which the proportionate share of Y would increase in consequence of an increase in the quantity of Y due to a shift in Y's maintenance function. Hence, provided that X's elasticity of maintenance and Y's elasticity of maintenance are *both* positive or negative on a backward-rising curve, or are *both* negative on a forward-falling curve, an increase in X must affect X's proportionate share in the same sense as that in which an increase in Y would affect Y's proportionate share. But, if one of the elasticities of maintenance is positive or negative on a backward-rising curve, and the other negative on a forward-falling curve, the opposite of this is true.[2]

§ 18. If the maintenance conditions of X and Y are interdependent,

$$\left\{1-\frac{dy}{dx}\right\}=\frac{{}_y\epsilon_y\left(1+\frac{{}_ye_y}{{}_xe_y}\right)}{{}_y\epsilon_y-{}_ye_y}$$

Provided that ${}_ye_y/{}_xe_y$ is either positive, or, if negative, is numerically <1, the presence of this element does not affect the sign of the expression $\left\{1-\frac{dy}{dx}\right\}$, and, therefore, leaves the conclusion of the preceding section intact. If, however, ${}_ye_y/{}_xe_y$ is negative and numerically >1, its presence reverses the sign of $\left\{1-\frac{dy}{dx}\right\}$, and therewith reverses also all the conclusions of § 16.

[1] Cf. my article on the "Elasticity of Substitution" in the *Economic Journal*, June 1934.

[2] Cf. Kahn, "Note on Elasticity of Substitution," *Review of Economic Studies*, October 1933, p. 77.

§ 19. In the special case where the quantity of the second factor Y is fixed, that is to say where ${}_ye_y = 0$, the condition for X's proportionate share increasing with an increase in its quantity is in all circumstances that

$$\left\{1 + \frac{1}{{}_x\epsilon_x} + \frac{1}{{}_y\epsilon_y}\right\}$$

is positive. We have then

$$\frac{1}{{}_y\epsilon_y} = -\frac{1}{{}_x\epsilon_y} = -\frac{dp_y}{dx}\cdot\frac{x}{p_y}.$$

If, y being fixed, we write $g(x)$ for total output, we have

$$p_y = \frac{g(x) - xg'}{y}.$$

$$\therefore \qquad \frac{dp_y}{dx} = \frac{xg''}{y} - \frac{dy}{dx}(g - xg')\frac{1}{y^2}.$$

But $\dfrac{dy}{dx} = 0$.

$$\therefore \frac{dp_y}{dx} = -\frac{xg''}{y}:$$

which, our units being such that x and y are equal, $= -g''$.

$$\therefore \frac{1}{{}_y\epsilon_y} = \frac{xg''}{p_y} = \frac{xg''}{g'}\cdot\frac{g'}{p_y} = \frac{1}{{}_x\epsilon_x}\cdot\frac{p_x}{p_y}.$$

$$\therefore \left\{1 + \frac{1}{{}_x\epsilon_x} + \frac{1}{{}_y\epsilon_y}\right\} = 1 + \frac{1}{{}_x\epsilon_x}\left(1 + \frac{p_x}{p_y}\right).$$

This is positive or negative according as $-\dfrac{1}{{}_x\epsilon_x}$ is $>$ or $< \dfrac{p_x + p_y}{p_y}$.

Therefore the condition for X's proportionate share of the product increasing as its quantity increases is that ${}_x\epsilon_x$ is numerically $> \dfrac{p_x + p_y}{p_y}$. This is equivalent to the formula given in the footnote to *The Economics of Welfare*, p. 665.

Finally, if we add the further condition that the productivity function in respect of X is linear, it follows that ${}_x\epsilon_x = \dfrac{2p_y}{p_x}$. Hence in this case the condition for X's proportionate share of the product increasing when X increases is that $\dfrac{2p_y}{p_x} > \dfrac{p_x + p_y}{p_y}$: *i.e.* that $(p_y - p_x)(2p_y + p_x)$ is positive. This means that initially X's absolute share of the product was less than Y's absolute share.

XII

Appendix to Chapter XL

§ 1. To prove that a constant elasticity demand curve and a constant elasticity supply curve, which is positively inclined and has an elasticity > 1, must be convex towards one another for all regions where the demand curve lies above the supply curve.

Write E for elasticity. If then, $f(x)$ be the demand, or supply, price of x units of commodity, in respect of x units

$$\frac{1}{\mathrm{E}}=\frac{xf'}{f}.$$

The condition for this to be constant is that

$$\frac{d}{dx}\left\{\frac{xf'(x)}{f(x)}\right\}=0.$$

That is,

$$\frac{xf''}{f'}+1=\frac{xf'}{f}.$$

$\therefore$

$$xf''=f'\left(\frac{1}{\mathrm{E}}-1\right).$$

If the curve is a demand curve, E is negative; which implies that f' is negative.

$\therefore f''$ is positive; *i.e.* the curve is convex to the horizontal axis. If the curve is a supply curve, positively inclined with an elasticity > 1, E is positive; which implies that f' is positive. Also $\left(\frac{1}{\mathrm{E}}-1\right)$ is negative.

$\therefore f''$ is negative; *i.e.* the curve is concave to the horizontal axis.

Hence, when a constant elasticity demand curve and a constant elasticity supply curve of positive inclination and of an elasticity greater than unity are associated together, they must be convex to one another for all regions where the demand curve lies above the supply curve.

§ 2. To prove that output in a single industry, provided that

the centres of production are of optimum size, is always smaller under the rule of monopoly than under the rule of competition.

Let x be quantity of output, $\phi(x)$ demand price, and $F(x)$ total cost of production. In all circumstances the net receipts of the hirer-seller

$$=\{x\phi(x)-F(x)\}.$$

Therefore the rate at which these net receipts increase when x increases

$$=\phi(x)\left\{1+\frac{x\phi'(x)}{\phi(x)}\right\}-F'(x).$$

If we write η for the elasticity of demand in respect of output x, this becomes

$$\left[\phi(x)\left\{1+\frac{1}{\eta}\right\}-F'(x)\right].$$

Provided that the centres of production are of optimum size, the output proper to competition is given by the equation

$$\phi(x)=\frac{F(x)}{x}.$$

Write e for the elasticity of supply. Then

$$\frac{1}{e}=x\frac{\dfrac{d}{dx}\left\{\dfrac{F(x)}{x}\right\}}{\dfrac{F(x)}{x}}=x\frac{F'(x)}{F(x)}-1.$$

$$\therefore \qquad F'(x)=\left(1+\frac{1}{e}\right)\frac{F(x)}{x}.$$

Hence our fundamental expression

$$\left\{\phi(x)\left\{1+\frac{1}{\eta}\right\}-F'(x)\right\}$$

may be written

$$\left\{\phi(x)\left\{1+\frac{1}{\eta}\right\}-\frac{F(x)}{x}\left(1+\frac{1}{e}\right)\right\}.$$

But, as we have seen, for the output proper to competition

$$\phi(x)=\frac{F(x)}{x}.$$

Therefore, for that output, this reduces to

$$\phi(x)\left\{\frac{1}{\eta}-\frac{1}{e}\right\}.$$

Since η is always negative, if e is positive or nil, this is obviously

negative. If e is negative, it seems at first sight that it may be positive. But in Appendix XI. § 8 it was shown that—apart from the case of backward-rising curves, which obviously cannot occur in the present connection—outputs, in respect of which e has a smaller negative value than η, are unstable, and cannot be maintained. Hence, for all practicable situations under the rule of competition, e, if negative, must be numerically larger than η; which implies that $\left(\frac{1}{\eta}-\frac{1}{e}\right)$ is negative. That is to say, our fundamental expression

$$\left\{\phi(x)\left\{1+\frac{1}{\eta}\right\}-\mathrm{F}'(x)\right\}$$

must have a negative value in respect of the output proper to competition. This implies that output under monopoly is less than output under competition.

§ 3. In an industry where there is only one centre of production, it is impossible for the rule of competition, in the sense in which it is defined in Chapter XVI., to be followed. We cannot, therefore, compare the situation under the rule of monopoly with what would happen under that rule. It is, however, open to us, if we wish, to compare the situation under monopoly with what would happen if output were so regulated as to make demand price and supply price equal to one another. Supply price for this purpose is defined as follows. In respect of quantities of output for which marginal cost is greater than average cost, *i.e.* quantities larger than a centre of optimum size would produce, supply price is equal to marginal cost. But in respect of quantities of output for which marginal cost is less than average cost, *i.e.* quantities smaller than a centre of optimum size would produce, a rate of pay for factor-group units equal to marginal cost would entail these units receiving less than they could earn elsewhere; and the centre would, therefore, shut down. In respect of these quantities supply price is equal, not to marginal cost, but, as in a many-centre industry, to average cost. When this last condition rules in respect of the point where the demand and supply curves intersect, the relation between monopoly situations and situations in which demand price and supply price are equal is the same as that between monopoly situations and competitive situations, as described in the preceding section. Output under monopoly is smaller than output under an arrangement that equates demand price and supply price. When supply price is equal to marginal cost, equality between demand price and supply price entails, instead of the equality

$$\phi(x)=\frac{\mathrm{F}(x)}{x},$$

the new equality $\phi(x) = \mathrm{F}'(x).$

If output is established in conformity with this equality, our fundamental expression $\left[(\phi x)\left\{1 + \frac{1}{\eta}\right\} - \mathrm{F}'(x)\right]$, which measures the rate at which net receipts increase as x increases, reduces to $\eta\phi(x)$. Since η is negative, this is negative. Again, therefore, output under monopoly is less than output under an arrangement that equates demand price and supply price.

XIII

Appendix to Chapter XLI

§ 1. The extent to which hirer-sellers exploit factors of production in a system of completely symmetrical industries where the rule of monopoly is adopted everywhere.

In the conditions contemplated in Chapter XLI., all industries being alike, the quantity of factors engaged and the quantity of output in each industry are the same under all-round monopoly as under all-round competition.

Let the quantity of factor-group units employed in any industry be x, and the quantity of output $F(x)$.

Let $\phi\{F(x)\}$ be the demand price for x units of this quantity of output, and let units be so chosen that in the equilibrium proper to competition $F(x)=x$.

Let c be the rate of pay per factor-group unit under the rule of competition, mc the rate under the rule of monopoly. Then

$$\phi\{F(x)\}=c \quad . \quad . \quad . \quad . \quad \text{(I)}$$

$$\frac{d}{dx}\left\{F(x)\,.\,\phi\{F(x)\}-mcx\right\}=0. \quad . \quad . \quad \text{(II)}$$

Write η for the elasticity of demand. These equations then yield

$$m=\frac{xF'}{F}\left(1+\frac{1}{\eta}\right),$$

which, if conditions of constant return prevail, reduces to

$$m=\left(1+\frac{1}{\eta}\right).$$

Since η is negative, it follows, whether constant returns prevail or not, that, provided $-\eta>1$, m is larger, *i.e.* the degree of exploitation is less, the larger is $-\eta$.

If $-\eta=1, m=0$; if $-\eta<1$, m is negative. Since negative rates of remuneration for factors of production are obviously impossible, it follows that, where $-\eta<=1$, these rates (exclusive of the hirer-sellers) are reduced to nothing, and exploitation is absolute.

With constant returns, when $-\eta=\infty$, $m=1$, and there is no exploitation. With diminishing returns, since $\frac{xF'}{F}<1$, there must always be some exploitation. With increasing returns, since $\frac{xF'}{F}<1$, there need not be exploitation, even though $-\eta<\infty$.

§ 2. The elasticity of marginal productivity in terms of the average and the marginal (collective) product in any industry where all the centres of production are of optimum size.

Write E for the elasticity of average productivity, P for marginal collective, and p for average, product in respect of a situation where x factor-group units are engaged. Let $F(x)$ be total output of the industry. By definition

$$\frac{1}{E}=x\frac{d}{dx}\left\{\frac{F(x)}{x}\right\}\div\frac{F(x)}{x},$$

$$P=F'(x),$$

$$p=\frac{F(x)}{x}.$$

Hence
$$\frac{1}{E}=\left\{F'(x)-\frac{F(x)}{x}\right\}\div\frac{F(x)}{x}=\frac{xF'(x)}{F(x)}-1$$

$$=\frac{P}{p}-1.$$

$$\therefore\qquad \frac{P}{p}=\frac{1}{E}+1.$$

§ 3. The relation between elasticity of average productivity and elasticity of supply, when supply price is equal to average cost of production.

Write E, as above, for the elasticity of average productivity and e for the elasticity of supply. If x be the quantity of factor-group units and $F(x)$ total output, we already have

$$\frac{1}{E}=\frac{xF'(x)}{F(x)}-1.$$

Now
$$\frac{1}{e}=F(x)\cdot\frac{d}{dF(x)}\left\{\frac{x}{F(x)}\right\}\div\frac{x}{F(x)}$$

$$=\frac{F(x)}{xF'(x)}-1.$$

$$\therefore\qquad \frac{1}{e}+1=\frac{1}{\frac{1}{E}+1}.$$

XIV

Appendix to Chapter XLIV

§ 1. The twofold aspect of the earnings of factor-group units in a particular centre of production in an industry where factor-group units are perfectly mobile.

When there is complete freedom of movement for factor-group units, their rate of pay must be the same in any centre of any industry as it is in other centres and other industries. For simplicity of exposition—the substance of the argument is not affected—let us assume that all the centres are exactly alike. In these conditions we may be tempted to write $\phi(x)$ for the demand price of x units of the commodity, as delivered from any one centre, and $F(x)$ for the total expenses to the centre of making this output—the price of the factors being reckoned as equal to what rules elsewhere. Then, whether the centre acts monopolistically or not, we have one equation

$$x\phi(x) = F(x) \quad . \quad . \quad . \quad . \quad \text{(I)}$$

If the centre does not act monopolistically, we have, it seems, a second equation

$$\phi(x) = F'(x) \quad . \quad . \quad . \quad . \quad \text{(II)}$$

If it does so act, the second equation is,

$$\frac{d\{x\phi(x)\}}{dx} = F'(x) \quad . \quad . \quad . \quad . \quad \text{(III)}$$

In either case there are two equations and only one unknown. This must clearly be wrong. The solution, as indicated in § 4 of Chapter XLIV., is that the demand price for the output of each centre ought to have been expressed as a function, not of one variable, but of two. Thus write n for the number of centres. Our first equation then becomes

$$x\phi(x, n) = F(x) \quad . \quad . \quad . \quad \text{(IV)}$$

Our second equation, if the representative centre does not act monopolistically, becomes

$$\phi(x, n) = F'(x) \quad . \quad . \quad . \quad \text{(V)}$$

If the centre does act monopolistically, it becomes

$$\frac{dx\phi(x, n)}{dx} = \mathrm{F}'(x) \quad . \quad . \quad . \quad \text{(VI)}$$

In either event the two equations that confront one another contain not one, but two, unknowns.

If any centre—all being assumed to behave alike—does not act monopolistically, equations (IV) and (V) yield $\mathrm{F}'(x) = \frac{\mathrm{F}(x)}{x}$. That is to say, the output of the centre is such that marginal cost is equal to average cost of production.[1]

If the centre does not act monopolistically, equations (IV) and (VI) yield

$$\mathrm{F}'(x) = \frac{\mathrm{F}(x)}{x} + x\frac{d\phi(x, n)}{dx}.$$

If the element $\frac{d\phi(x, n)}{dx} = 0$, the solution is, of course, the same as the above. But, if this element is negative, $\mathrm{F}'(x) < \frac{\mathrm{F}(x)}{x}$; that is to say, marginal cost of production is less than average cost. Now for the element $\frac{d\phi(x, n)}{dx}$ to be nil means that the demand for supplies from the centre is perfectly elastic, in such wise that a very small rise in its price would destroy the whole of this demand. That cannot happen in a system of the kind considered here; for, in view of transport costs, the other centres cannot do more than partially supplant that centre. *A fortiori*, $\frac{d\phi(x, n)}{dx}$ cannot be positive. It follows that that element is in fact negative. Hence the marginal cost of production in every centre is in fact less than average cost; which implies, of course, that the centre is of less than optimum size.

An alternative formulation is as follows. Write $\mathrm{F}(p)$ for the total quantity of product that all the buyers together will buy if the price at works of all the centres is p. Let $f(p)$ be the quantity of product provided by each centre at average cost p. Let there be n exactly similar centres. Let $\frac{\partial\psi(p, n)}{\partial p}$ be the rate of substitution, among one centre's customers, of product from the other

[1] In the constructions commonly employed to show this geometrically two curves are drawn whose ordinates represent respectively average cost and marginal cost; the point of equilibrium being at the intersection of the curves. If we draw instead a single curve, whose ordinates measure total expenses of production, the point of equilibrium is that point in respect of which a tangent to the curve passes through the origin.

centres, when the price charged by the one centre, and by it alone, increases. Then, if the rule of monopoly is followed, the two fundamental conditions may be written

$$\frac{1}{n}F(p)=f(p) \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(I)}$$

$$\frac{1}{n}F'(p)-\frac{\partial\psi(p,n)}{\partial p}=f'(p) \quad . \quad . \quad . \qquad \text{(II)}$$

If there are no costs of transport, $\frac{\partial\psi(p,n)}{\partial p}$ is an infinite negative quantity.

$\therefore f'(p)$ is an infinite negative quantity.

$\therefore$ the actual value of p is the minimum average cost in respect of which it would pay our centre to have any output; *i.e.* the centre is of optimum size.

If there are some costs of transport, but not sufficient to isolate the markets completely from one another, $\frac{\partial\psi(p,n)}{\partial p}$ has a negative value less than infinite.

$\therefore$, since $F'(p)$ always has a finite negative value, $f'(p)$ has a finite negative value.

$\therefore$ the actual value of p is larger than the aforesaid average cost; and the centre is of less than optimum size.

If the costs of transport are large enough to isolate the markets completely, $\frac{\partial\psi(p,n)}{\partial p}=0$ for all values of p. In these circumstances our two conditions, apart from freak cases, can be jointly satisfied only for nil output. For practical purposes, though not, of course, in point of logic, they are incompatible.

§ 2. The relation between the actual and the optimum quantity of individual factors of production engaged in centres of less than optimum size.

Let the quantities of the several factors, exclusive of the hirer-seller, in a centre be x, y, z, and their prices there—equal to those ruling outside—a, b, c. Let output be $F(x, y, z)$ and average cost of product p.

Then
$$\frac{F(x,y,z)}{ax+by+cz}=\frac{1}{p}.$$

In order that p may be a minimum, and so $1/p$ a maximum,

$$\frac{\partial}{\partial x}\left\{\frac{F}{ax+by+cz}\right\}=0,$$

$$\frac{\partial}{\partial y}\left\{\frac{F}{ax+by+cz}\right\}=0,$$

$$\frac{\partial}{\partial z}\left\{\frac{F}{ax+by+cz}\right\}=0.$$

Hence
$$p\frac{\partial F}{\partial x}=a,$$
$$p\frac{\partial F}{\partial y}=b,$$
$$p\frac{\partial F}{\partial z}=c.$$

If $\dfrac{F(x, y, z)}{ax+by+cz}$ is less than a maximum, it is necessary either that $p\dfrac{\partial F}{\partial x}<a$, or that $p\dfrac{\partial F}{dy}<b$, or that $p\dfrac{\partial F}{\partial z}<c$.

But in any event, whether the centre is of optimum size or not, it is to the interest of the hirer-seller to make

$$\left\{p\frac{\partial F}{\partial x}\div a\right\}=\left\{p\frac{\partial F}{\partial y}\div b\right\}=\left\{p\frac{\partial F}{\partial z}\div c\right\}.$$

Therefore, if *any* of the inequalities $p\dfrac{\partial F}{\partial x}<a$ or $p\dfrac{\partial F}{\partial y}<b$ or $p\dfrac{\partial F}{\partial z}<c$ holds, all must hold.

That is to say, if the centre is of optimum size, the quantity of each factor engaged must be less than the quantity which, the quantities of the other factors being what they are, would make average cost a minimum.

INDEX

THE END

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